Light Fields in Ray and Wave Optics

Introduction to Light Fields:	Ramesh Raskar
Wigner Distribution Function to explain Light Fields:	Zhengyun Zhang
Augmenting LF to explain Wigner Distribution Function:	Se Baek Oh
Q&A	
Break	
Light Fields with Coherent Light:	Anthony Accardi
New Opportunities and Applications:	Raskar and Oh
Q&A:	All

CVPR 2009 Short Course

Light Fields: Present and Future (Computational Photography)

Using Wigner Distributions to Explain Light Fields

Zhengyun Zhang

Stanford University

IEEE Computer Society Conference on Computer Vision and Pattern Recognition 2009

Light Fields and Wave Optics

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Why Study Light Fields Using Wave Optics?

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macro

micro

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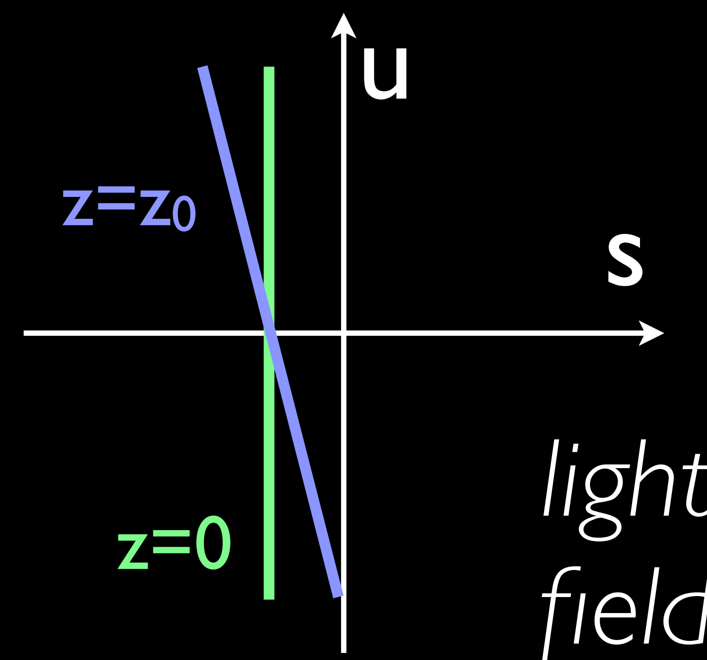

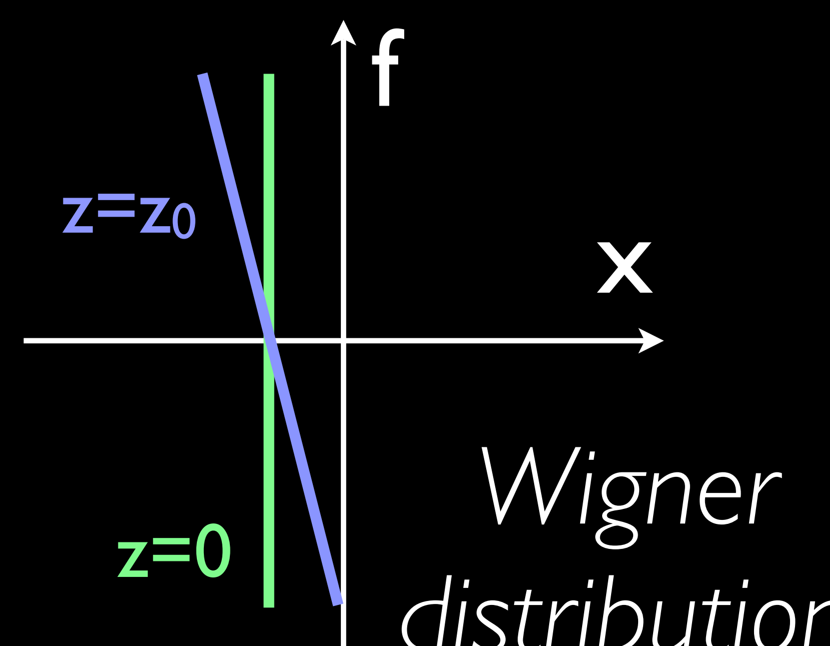

Outline

- review light fields and wave optics
- observable light field and
  the Wigner distribution
- applications

Light Fields

- radiance per ray
- ray parametrization:
  - position (s)
  - direction (u)

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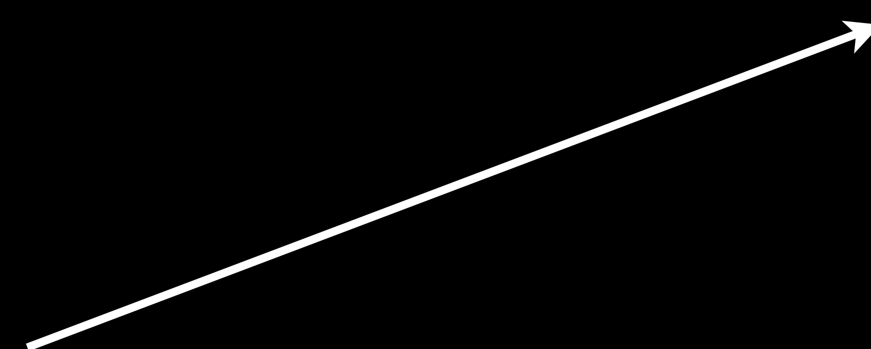

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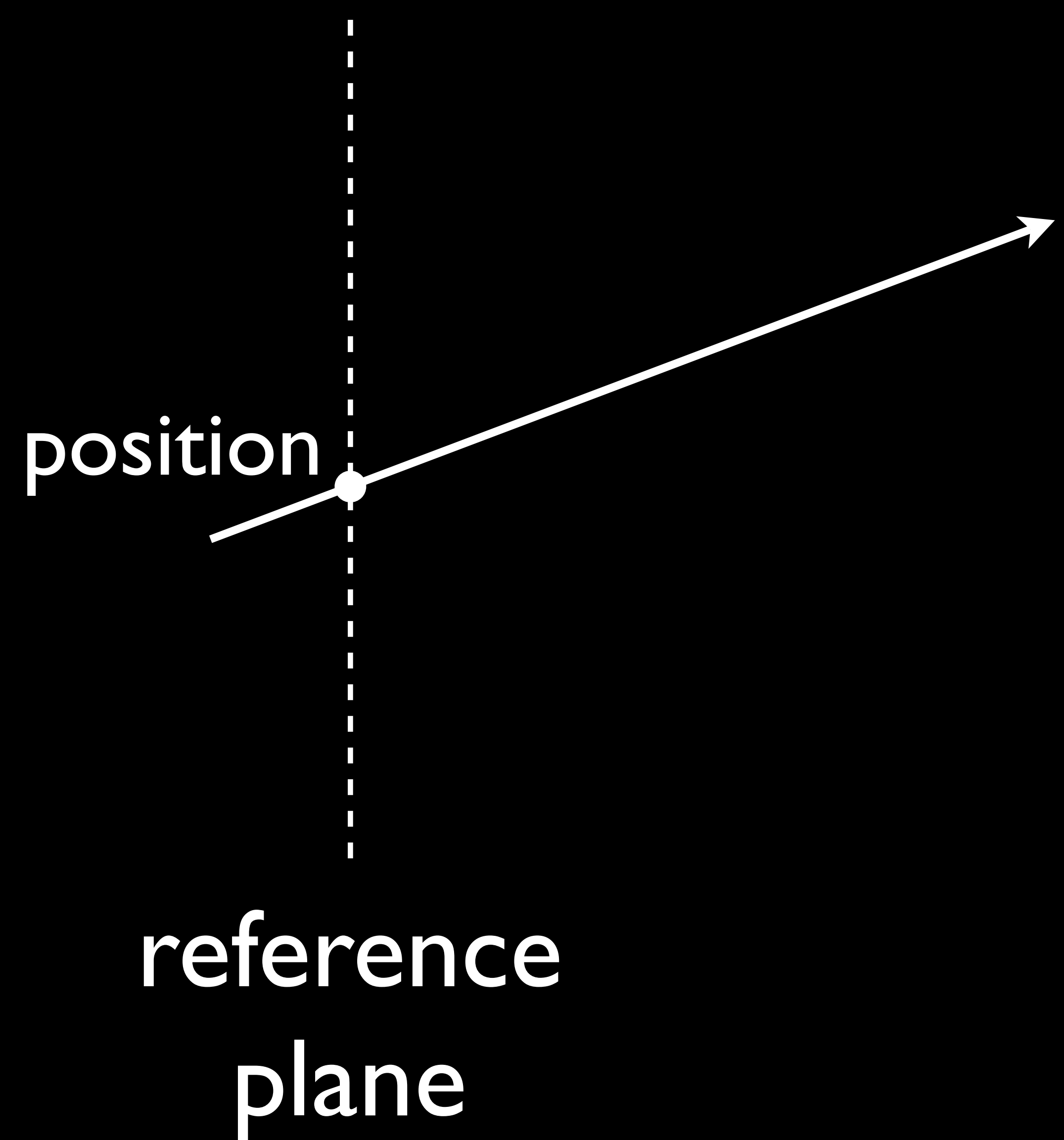

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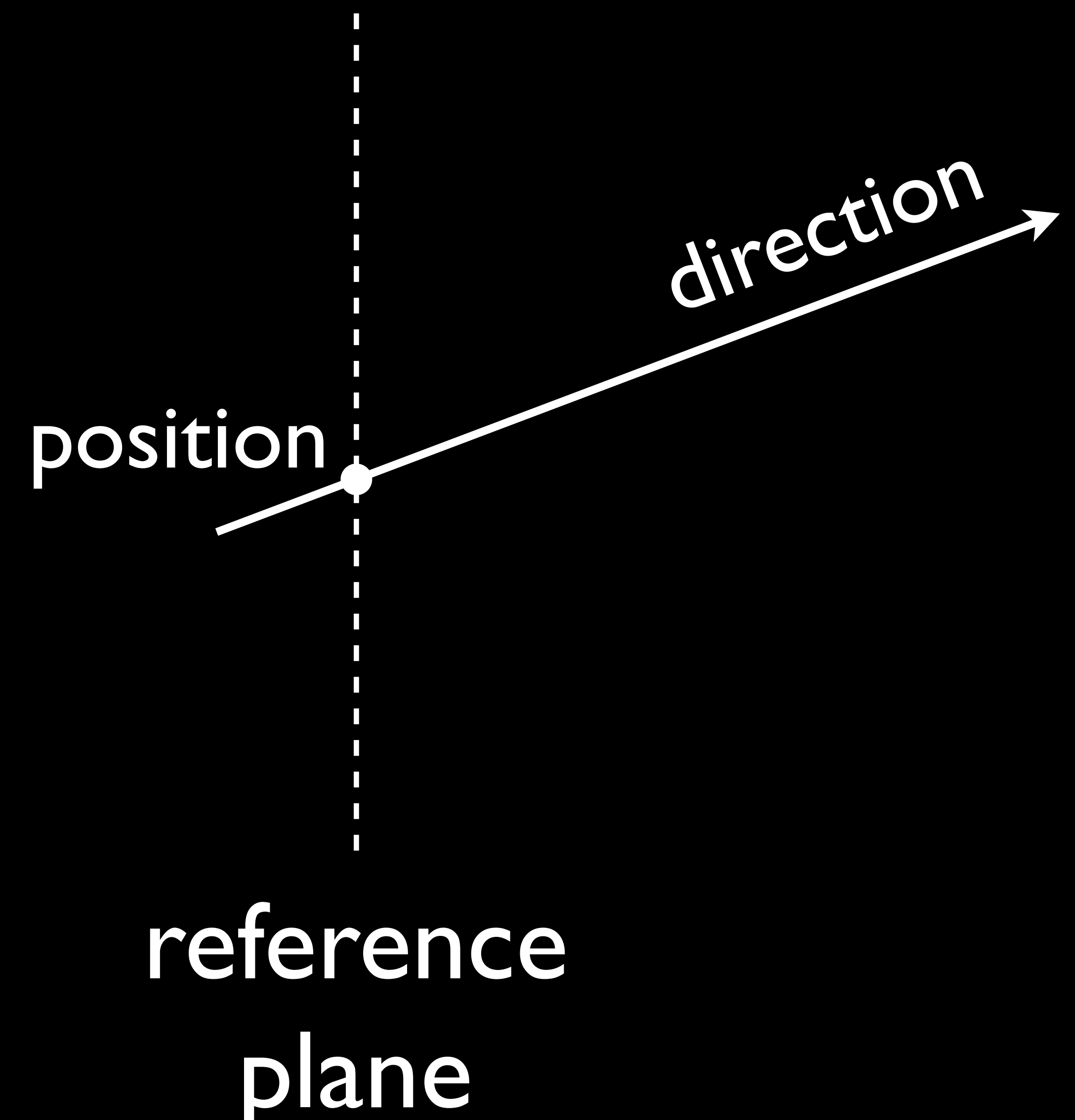
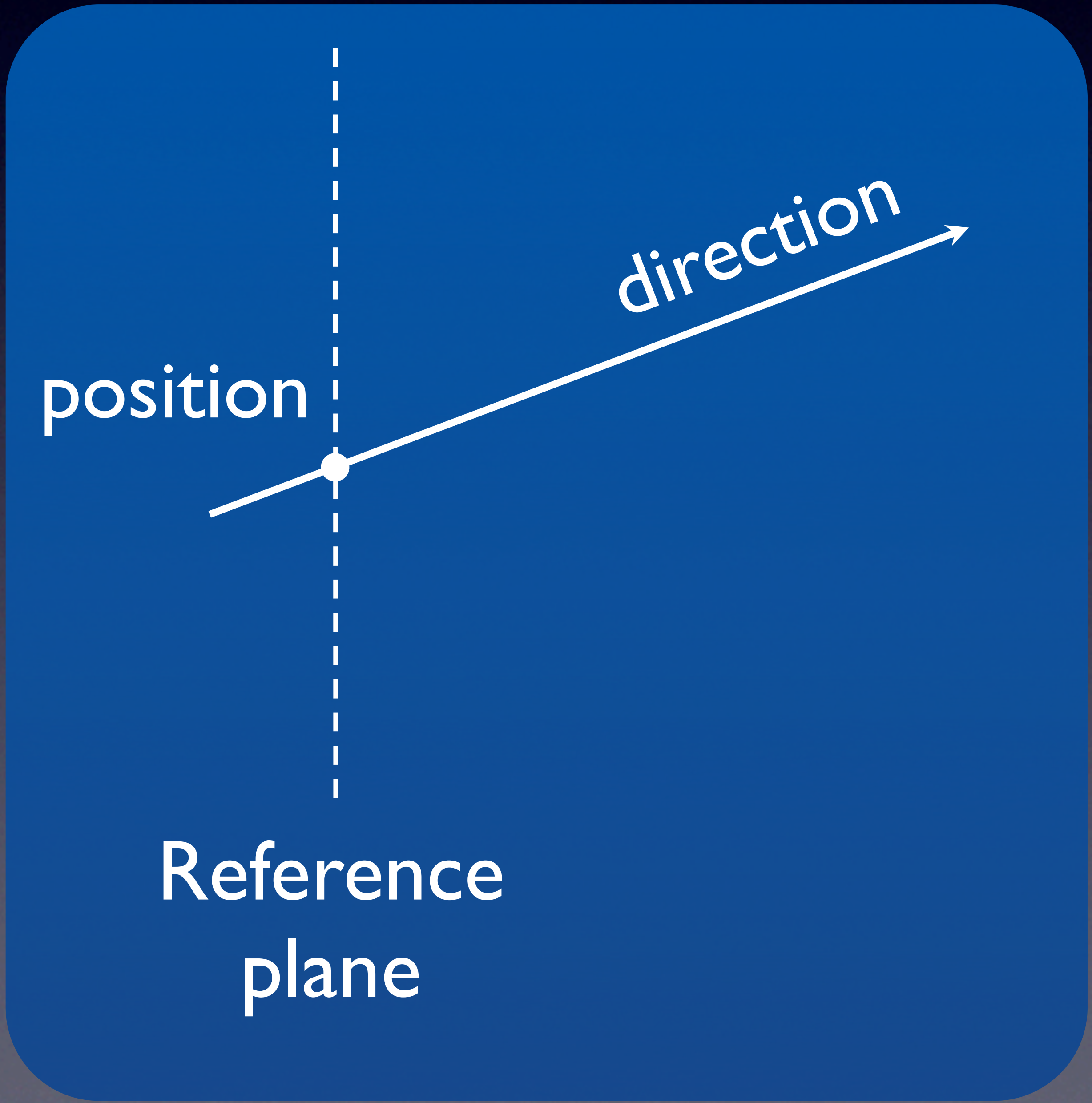


Light Fields

Goal: Representing propagation, interaction and image formation of light using purely position and angle parameters

- Radiance per ray
- Ray parametrization:
  - Position : s, x, r
  - Direction : u, θ, s



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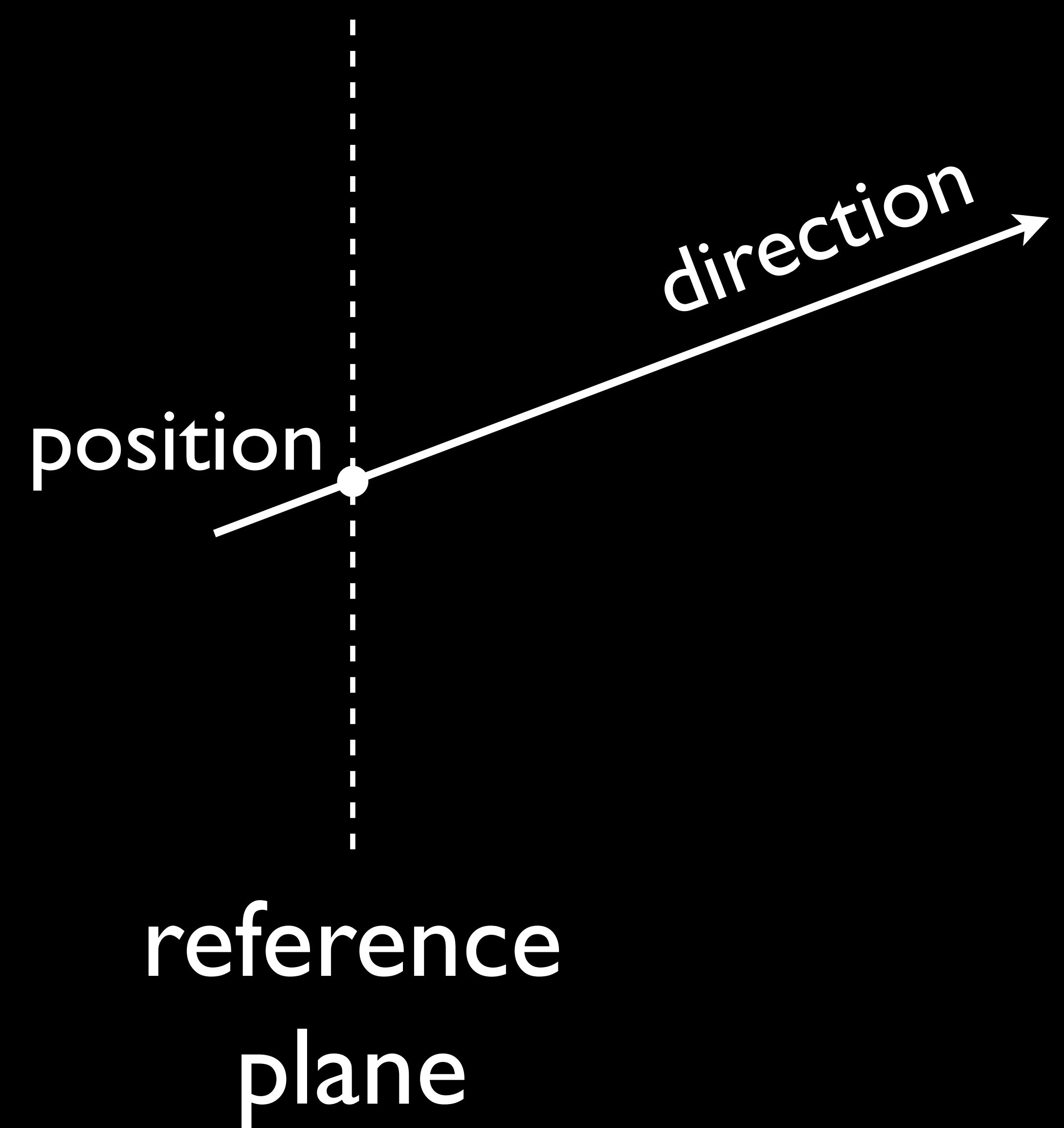
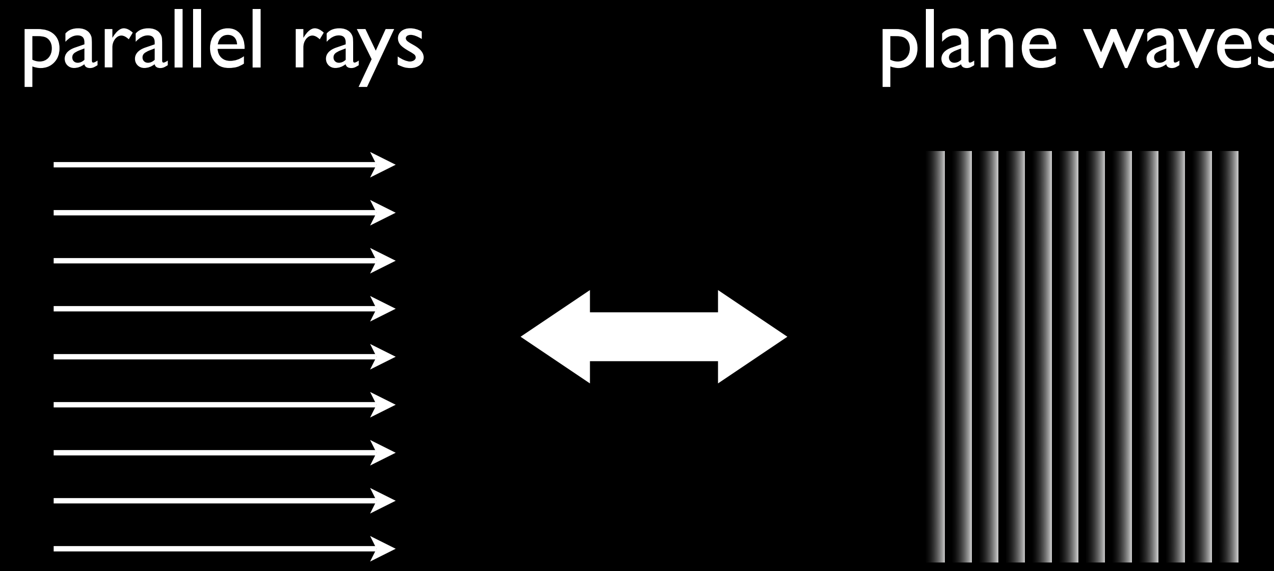


Wave Optics

- waves instead of rays
- interference, diffraction
- plane of point emitters (Huygen's principle)
- each emitter has amplitude and phase



(coherent and flatland)

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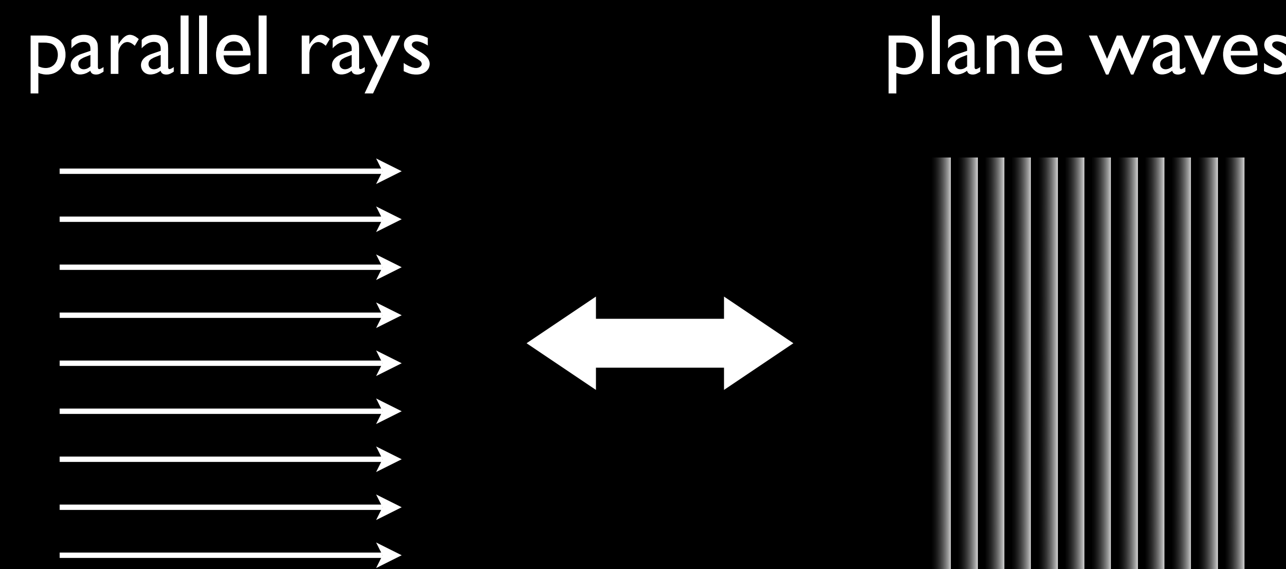

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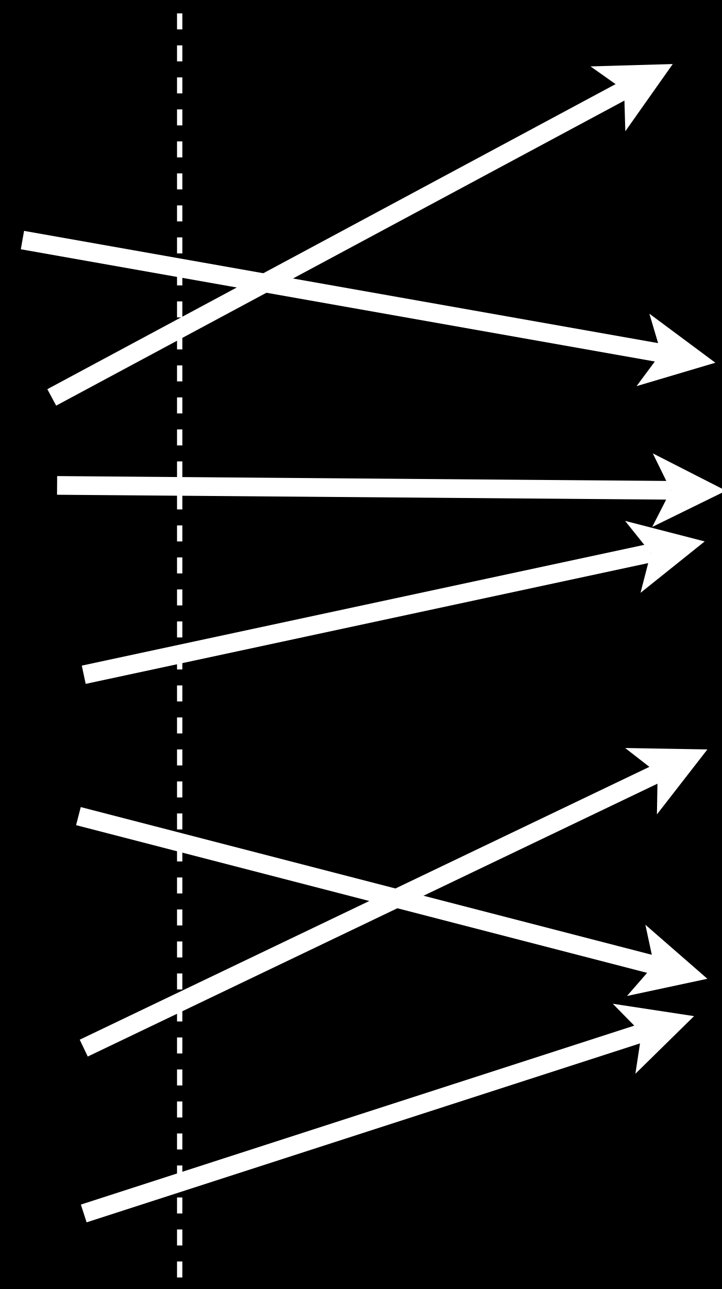

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$$U(x) = A(x)e^{j\phi(x)}$$

Position and Direction in Wave Optics

- recall: light field describes how power is spread over position and direction

$$U(x) = A(x)e^{j\phi(x)}$$

- point emitters on plane have amplitude and phase
- positional spread is amplitude squared

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$$I(x) = \left|A(x)e^{j\phi(x)}\right|^2$$

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$$U(x) = A(x)e^{j\phi(x)}$$

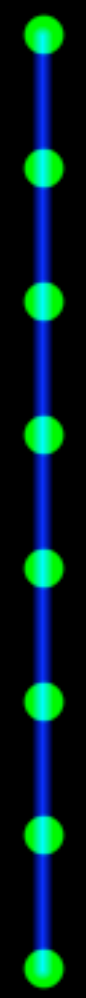
$$I(x) = A^2(x)$$

Position and Direction in Wave Optics

- direction
  - **axial**
  - oblique
  - more oblique

Position and Direction in Wave Optics

- direction
  - **axial**
  - oblique
  - more oblique



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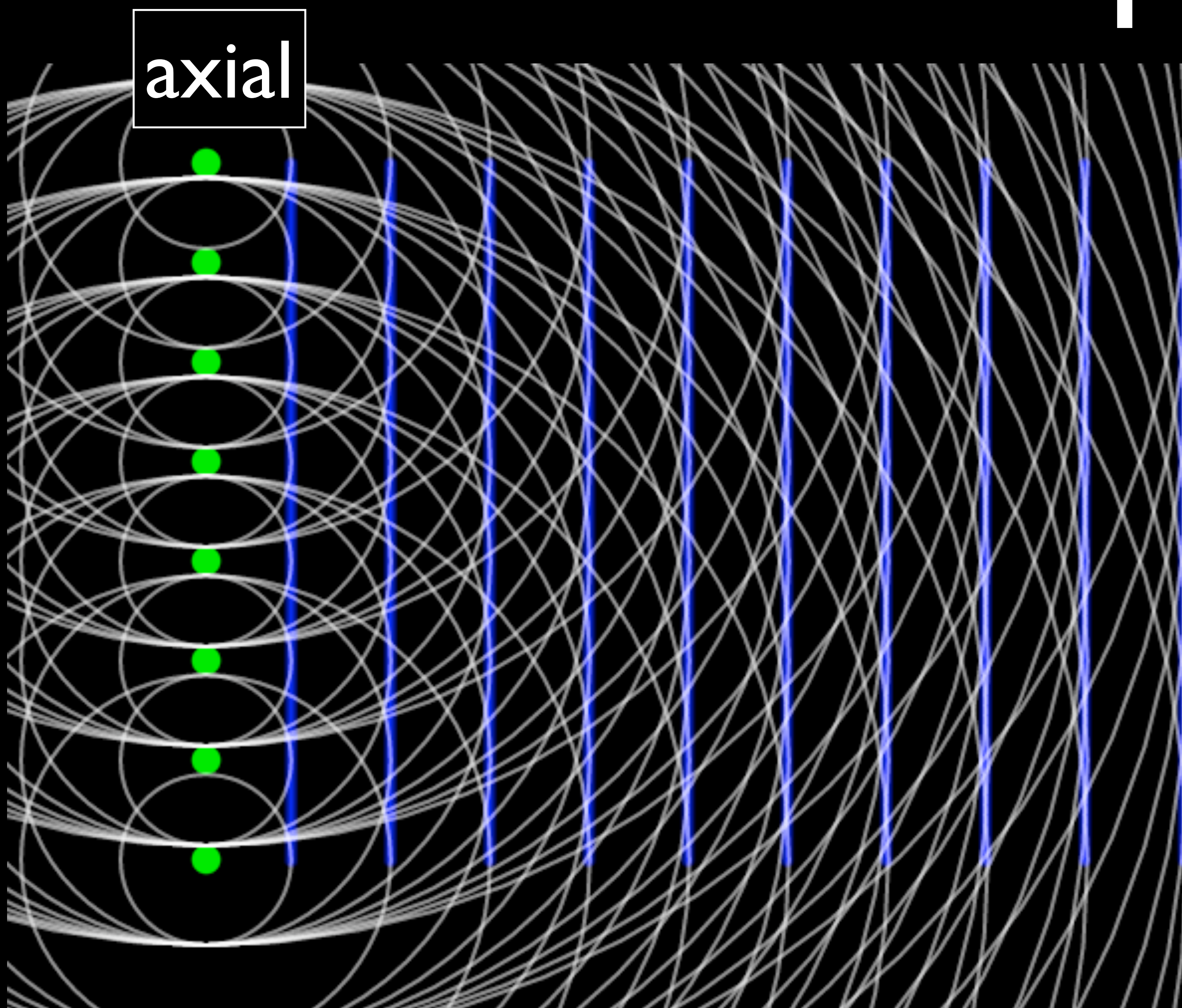
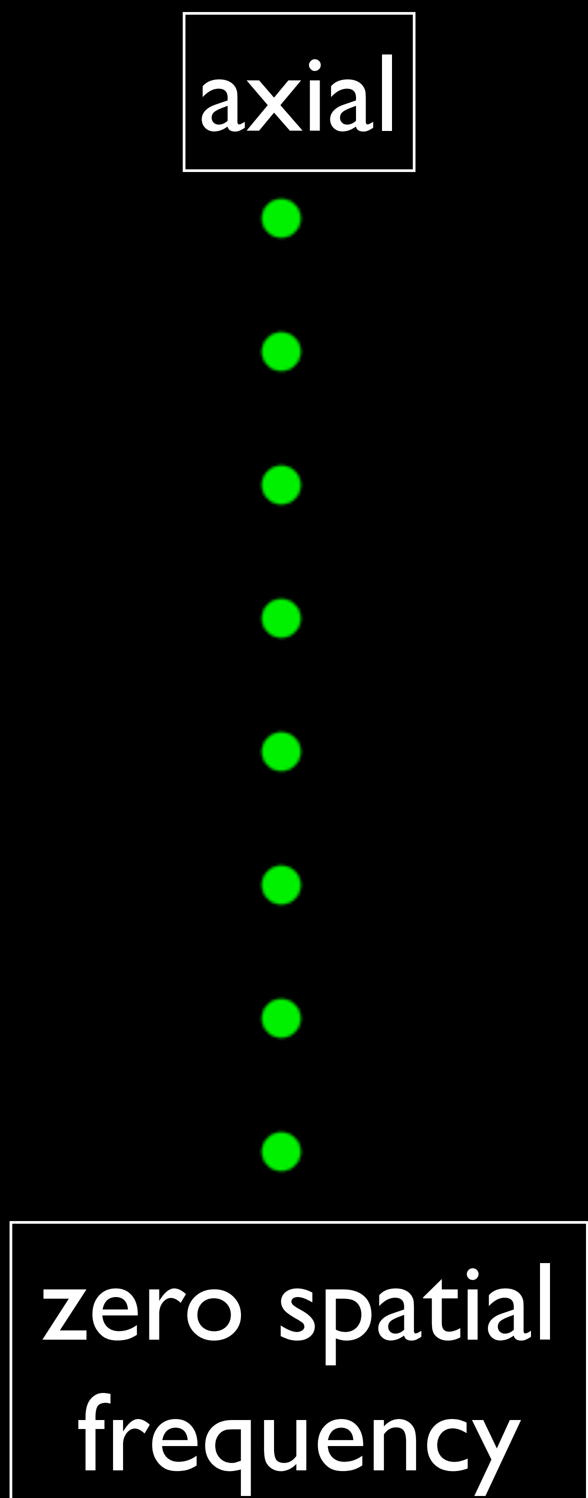


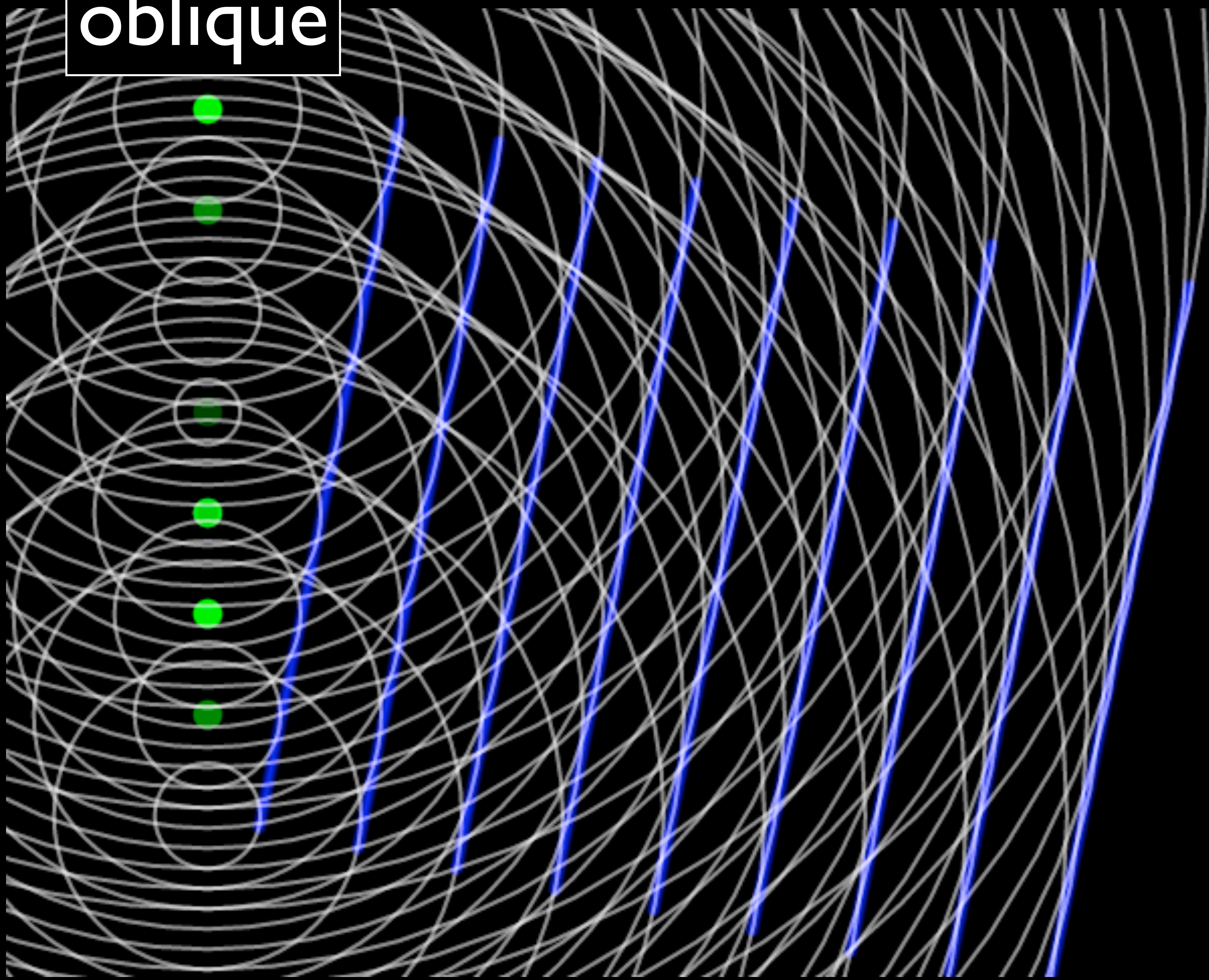
Position and Direction in Wave Optics



Position and Direction in Wave Optics

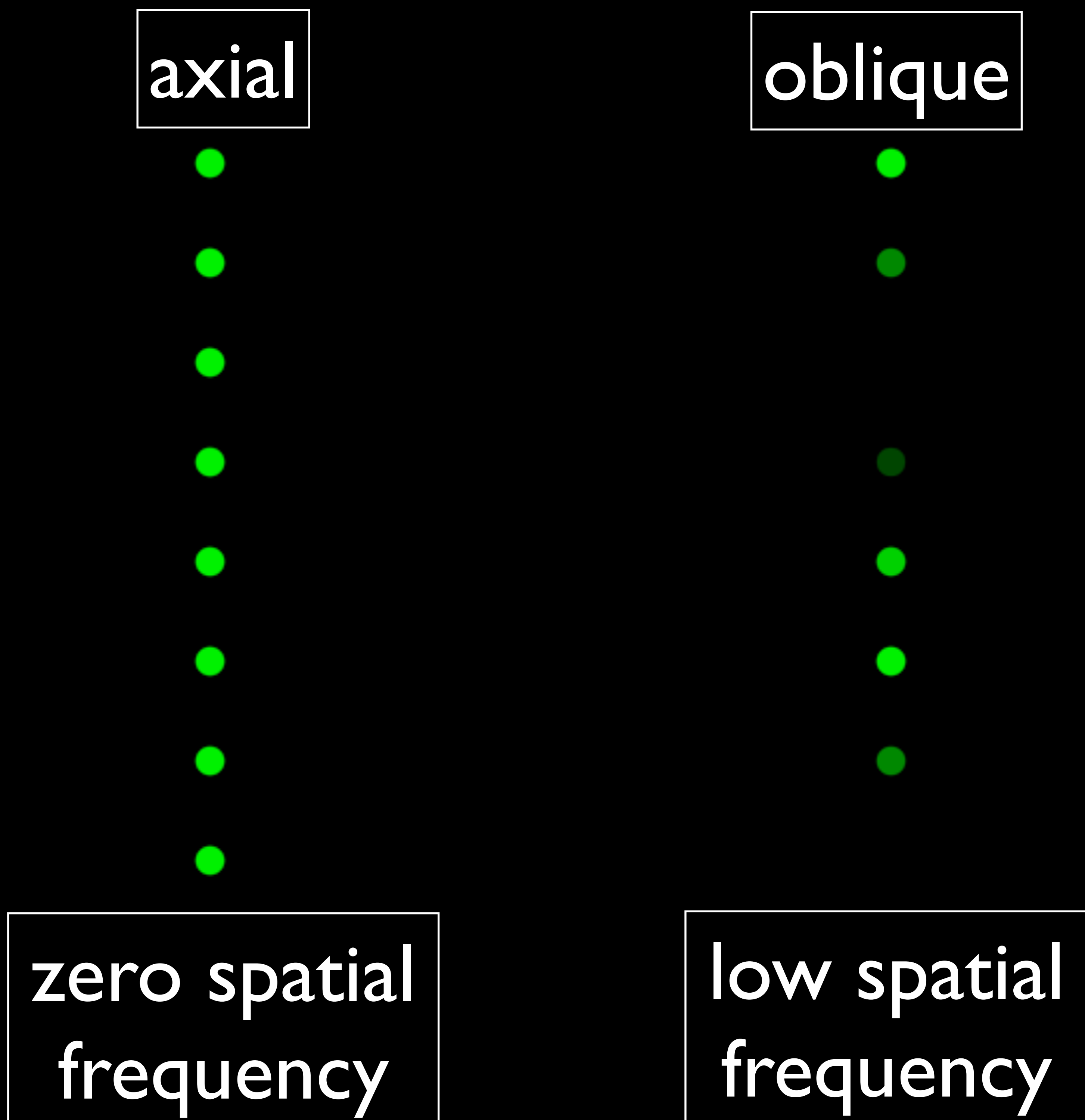
axial

oblique



zero spatial frequency

Position and Direction in Wave Optics



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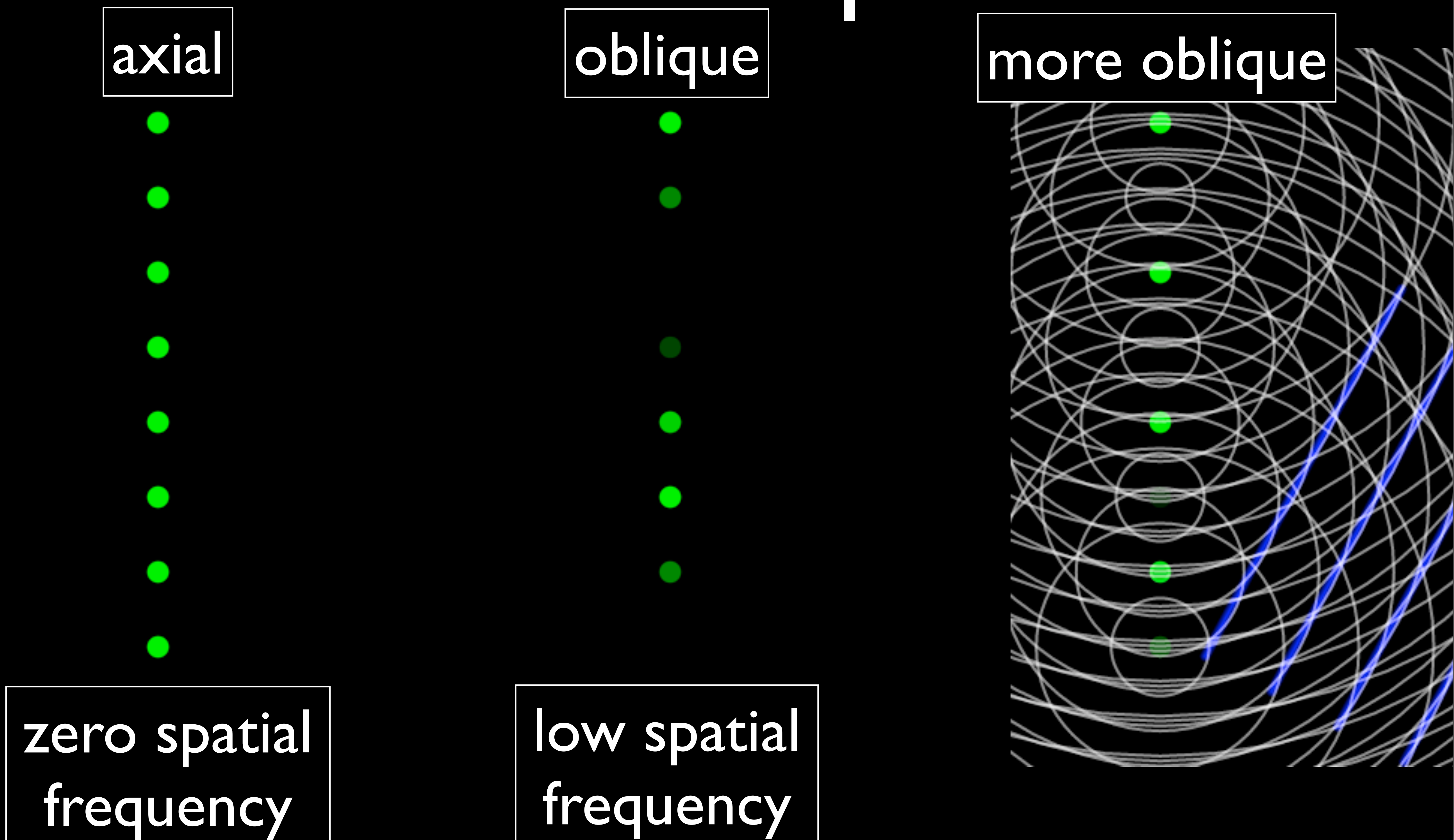

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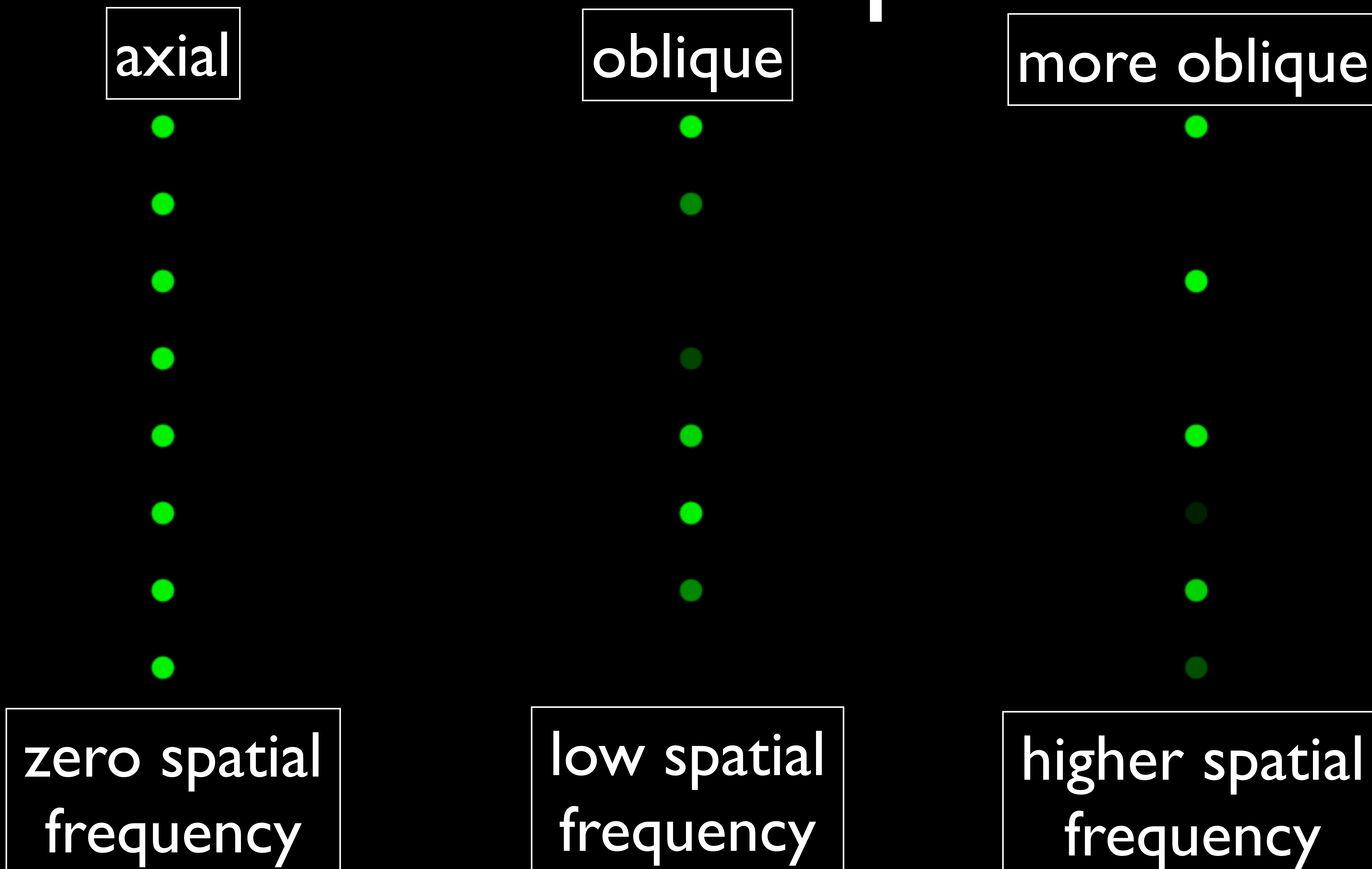

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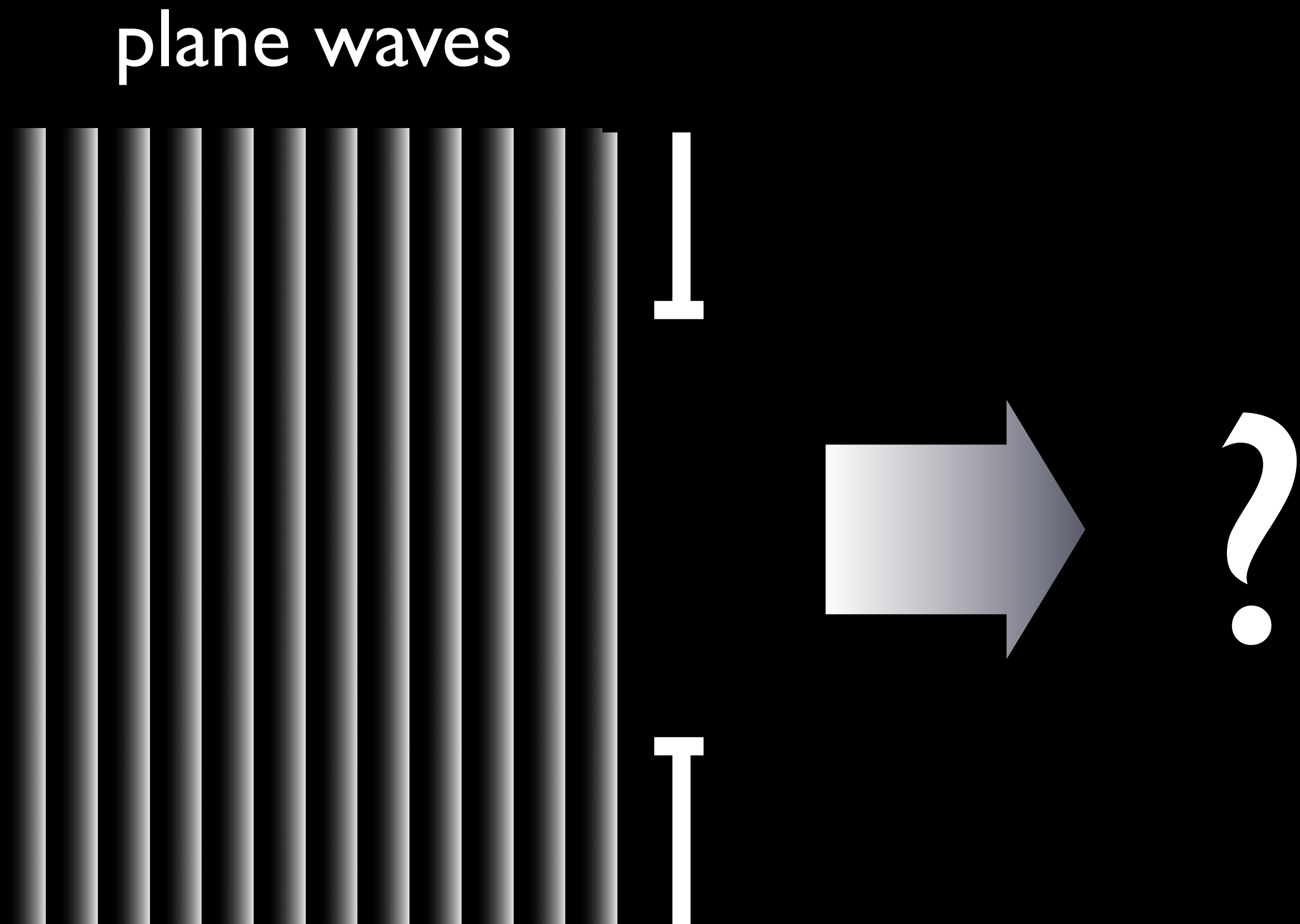

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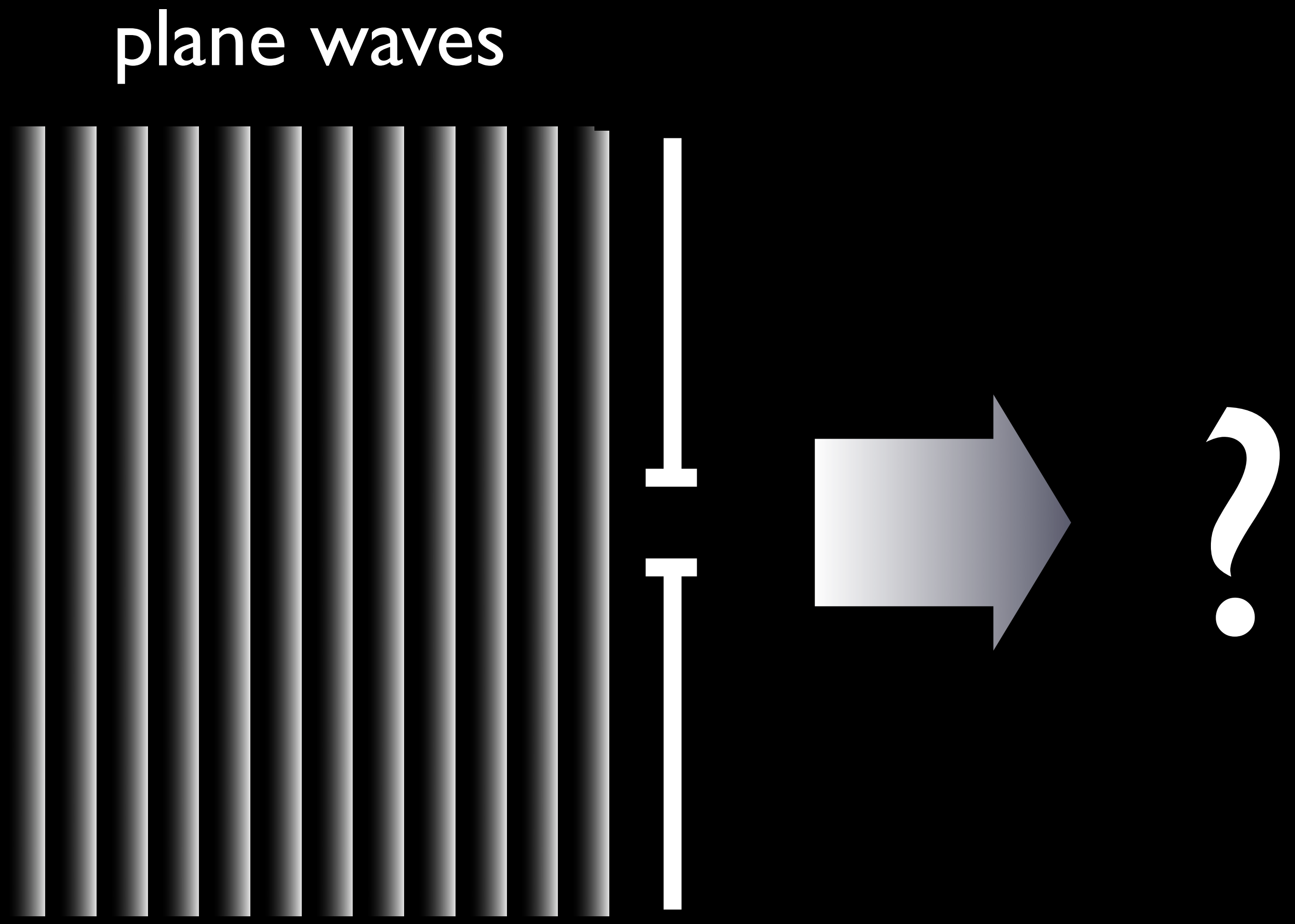

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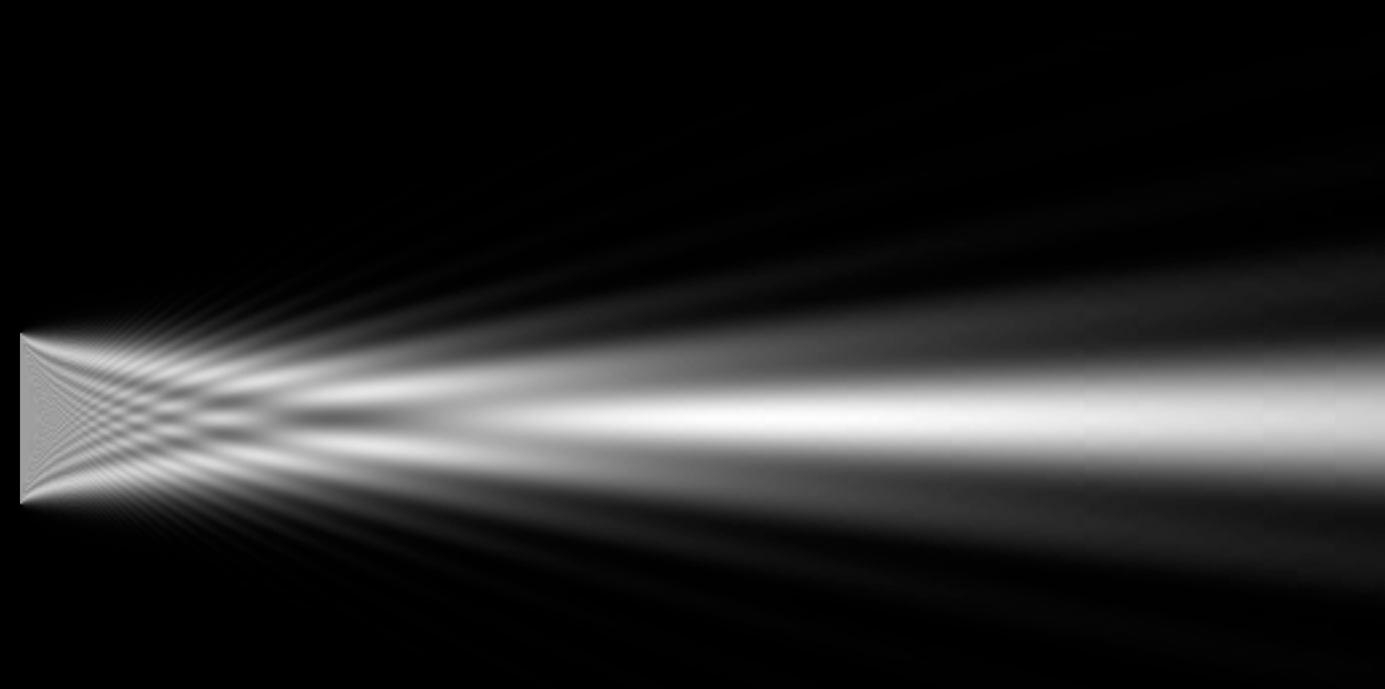


aperture = 128 wavelengths

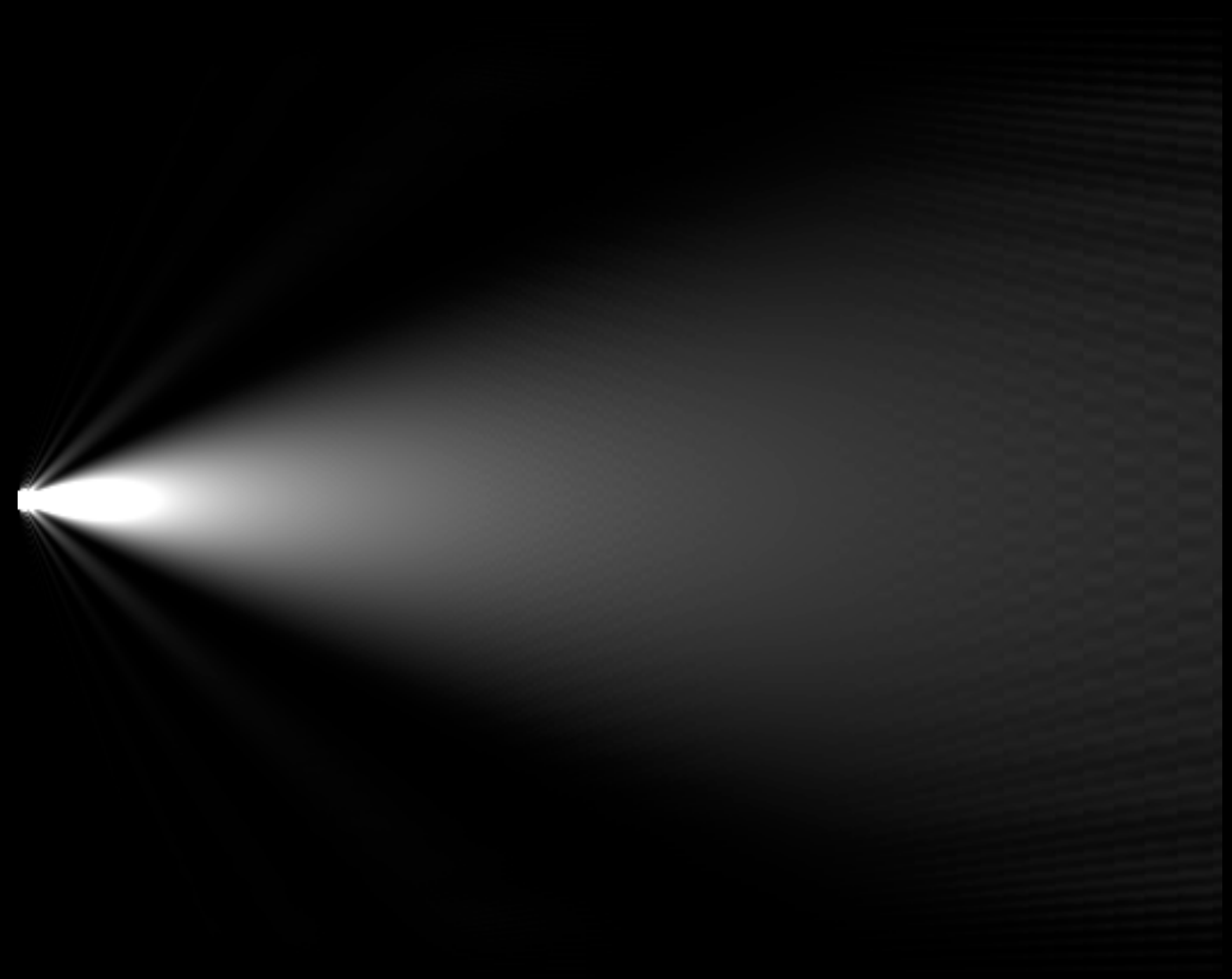
Position and Direction in Wave Optics

aperture = 64 wavelengths

Position and Direction in Wave Optics

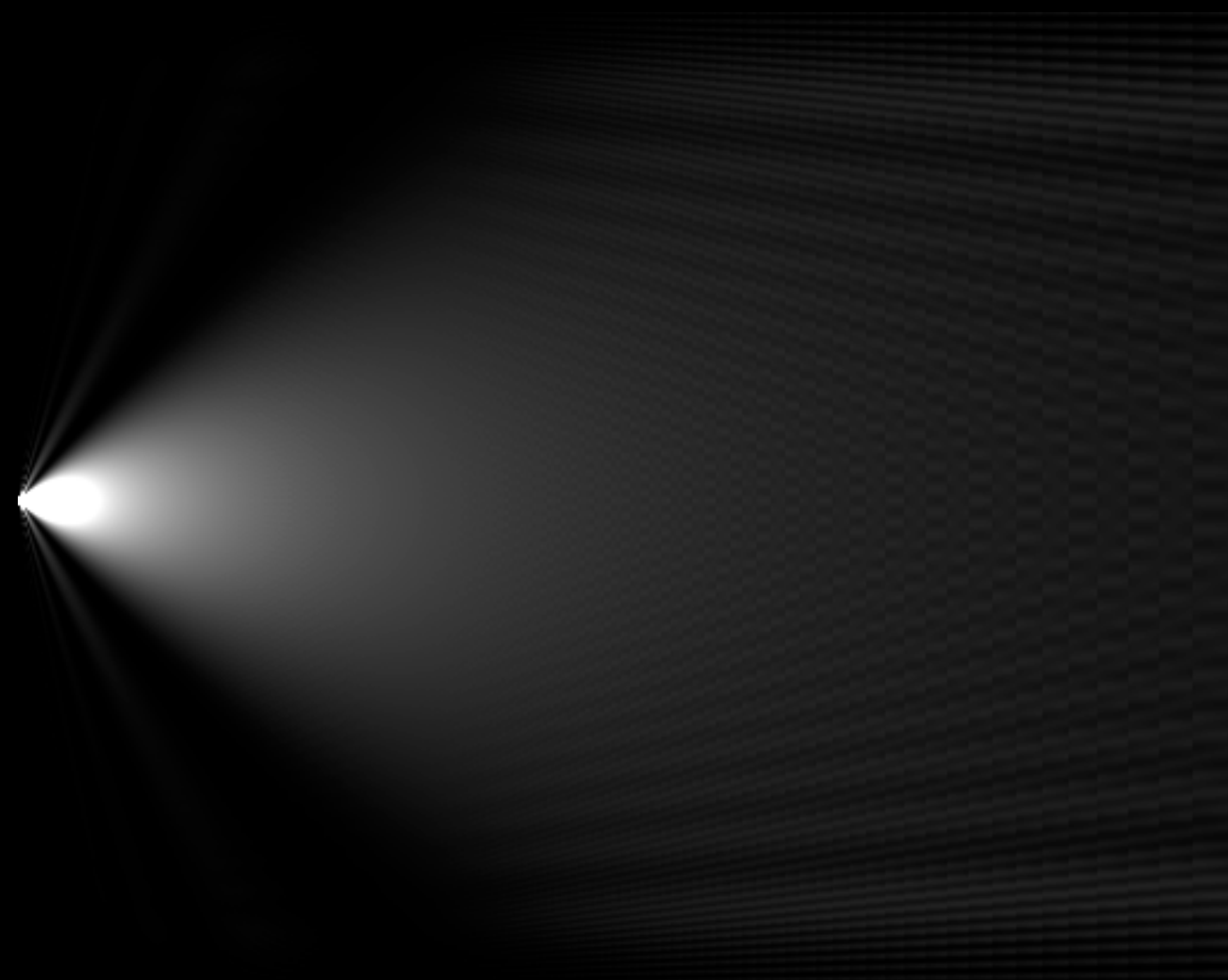
aperture = 32 wavelengths

Position and Direction in Wave Optics



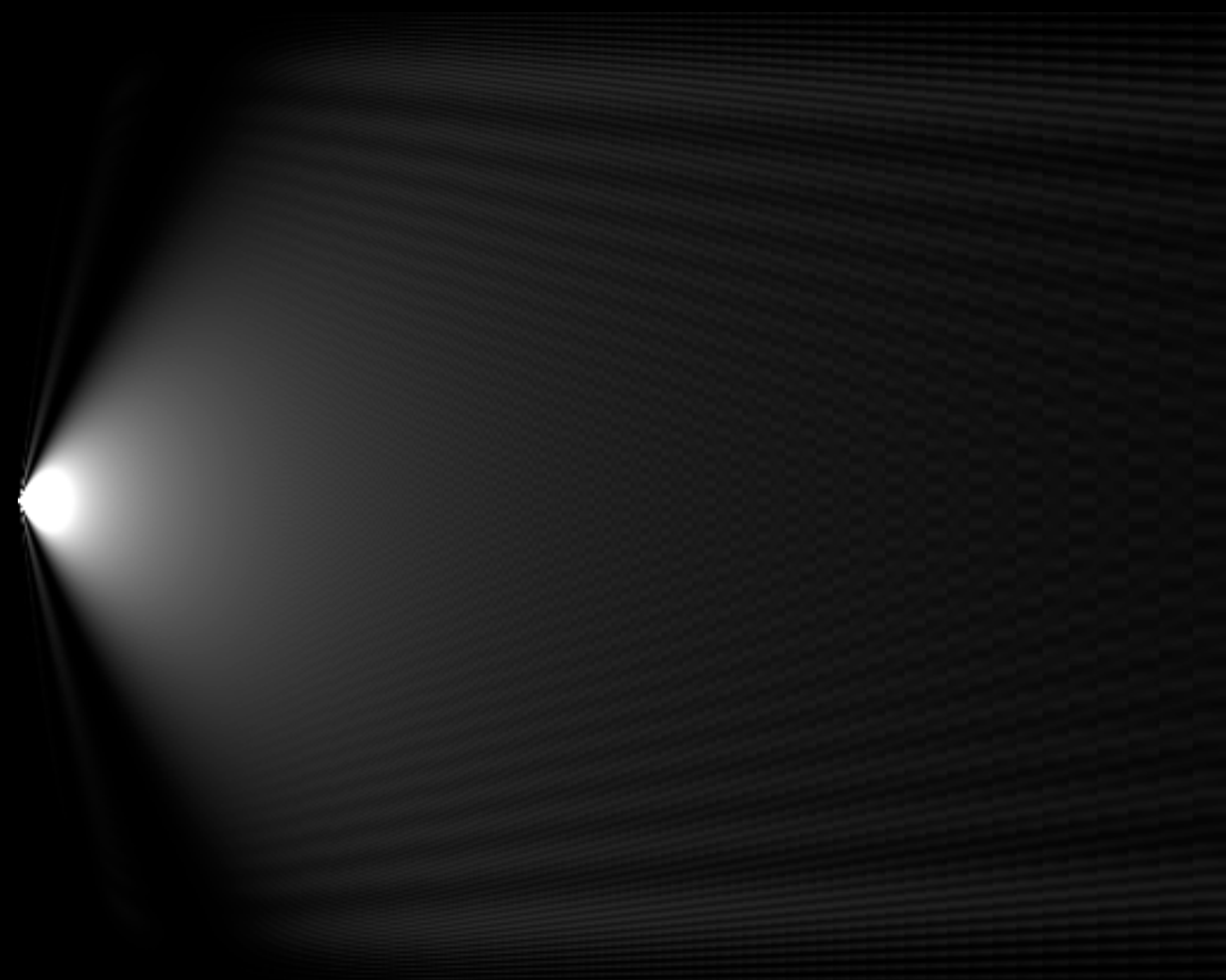
aperture = 16 wavelengths

Position and Direction in Wave Optics



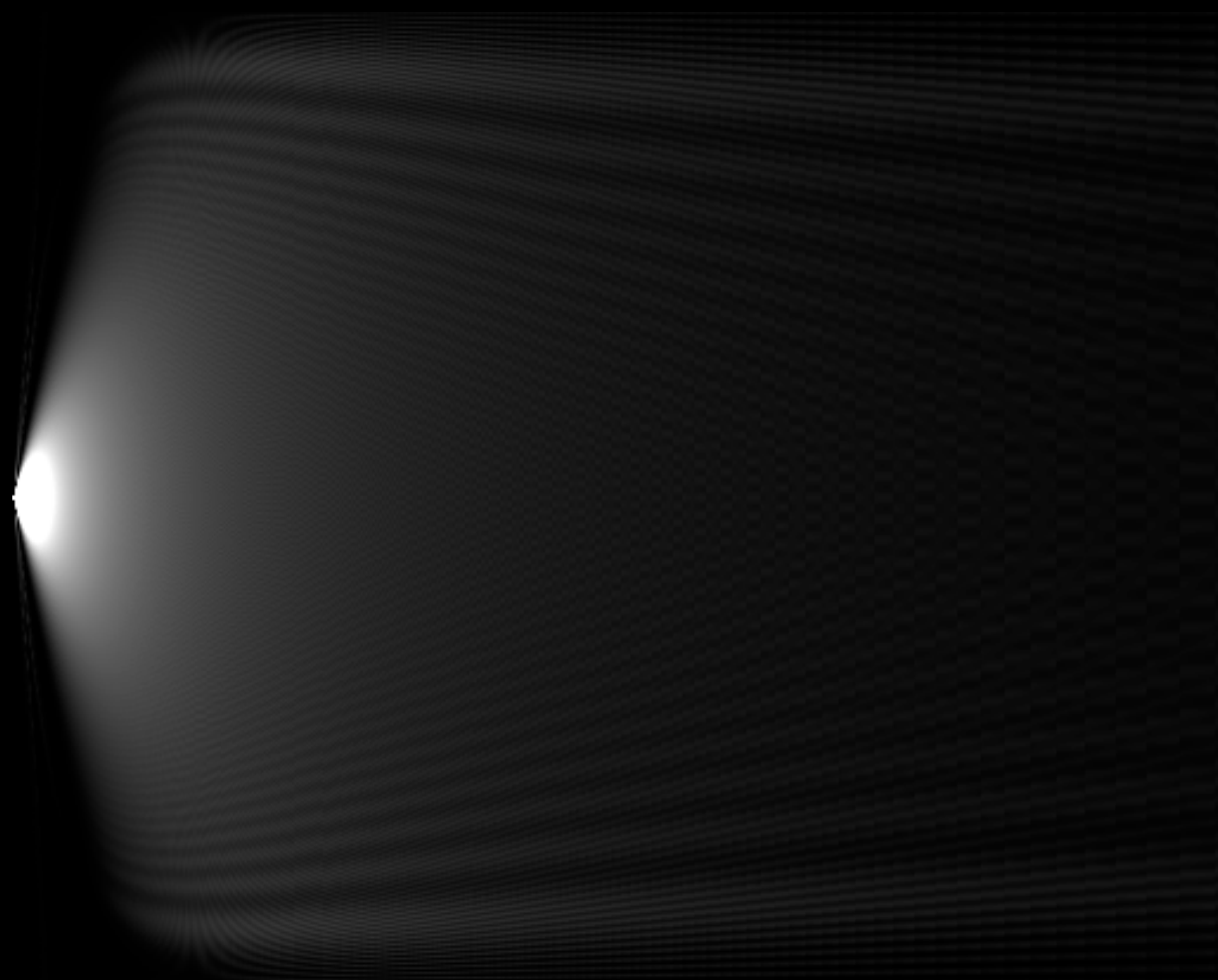
aperture = 8 wavelengths

Position and Direction in Wave Optics



aperture = 4 wavelengths

Position and Direction in Wave Optics



aperture = 2 wavelengths

Position and Direction in Wave Optics

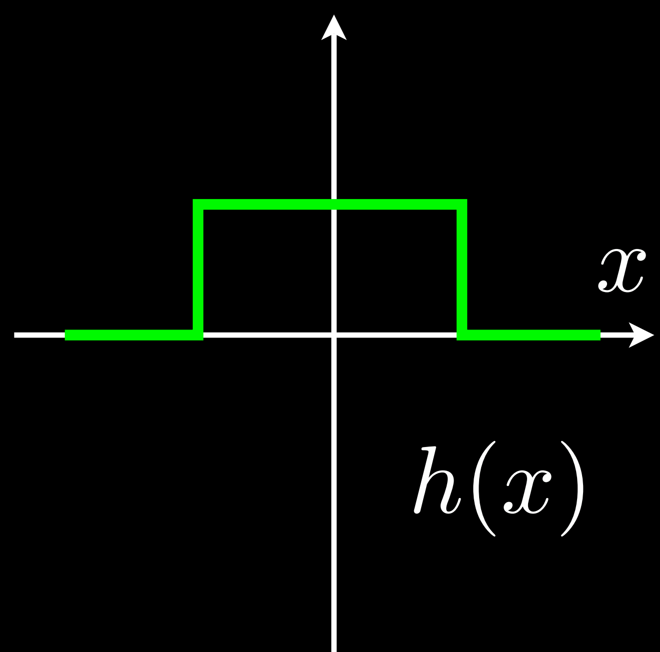
Recap

ray optics	position	direction
wave optics	position	spatial frequency

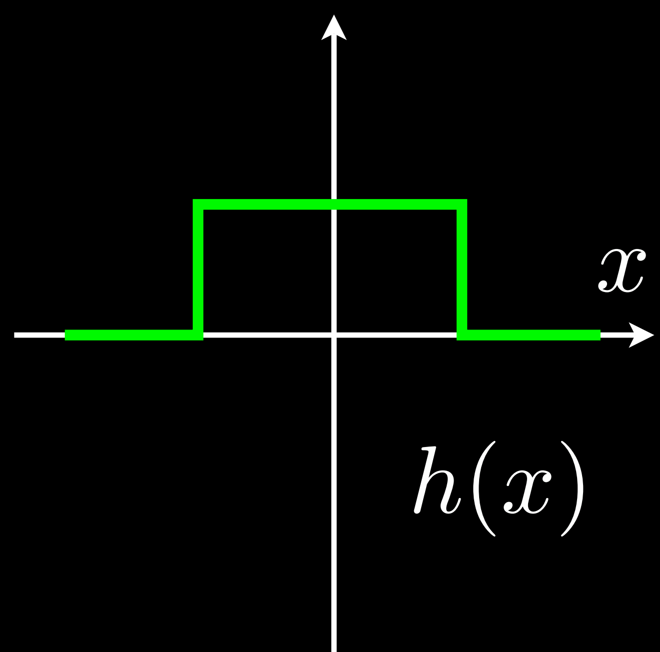
- to determine both position and spatial frequency, need to look at a window of finite (nonzero) width

2D Wigner Distribution

2D Wigner Distribution

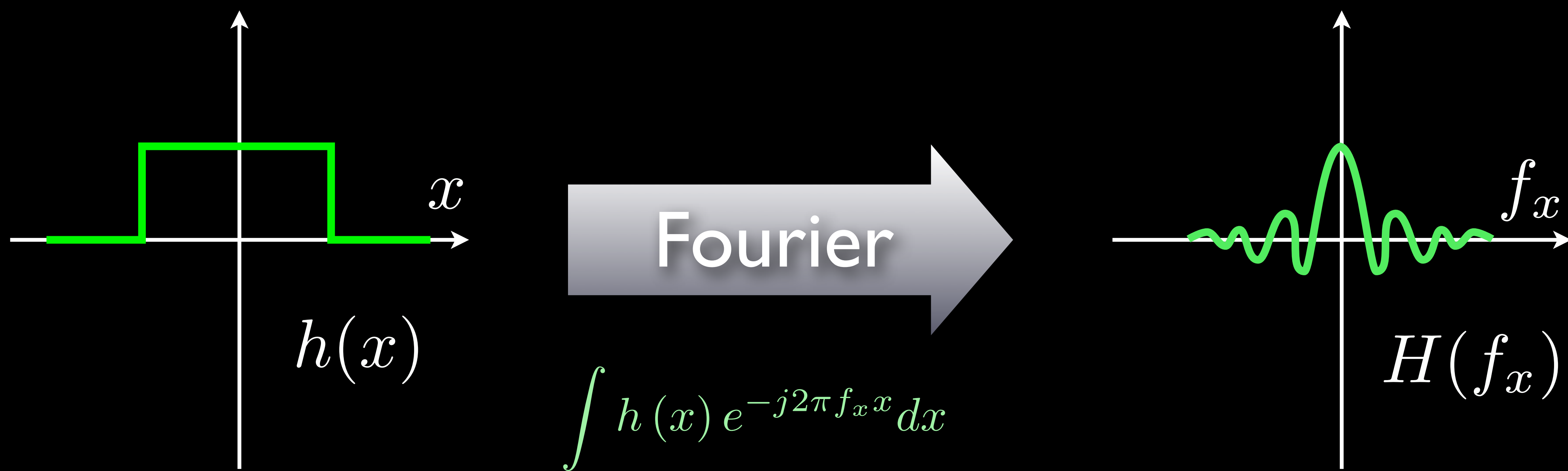


2D Wigner Distribution



$$\int h(x)\, e^{-j2\pi f_x x} dx$$

2D Wigner Distribution



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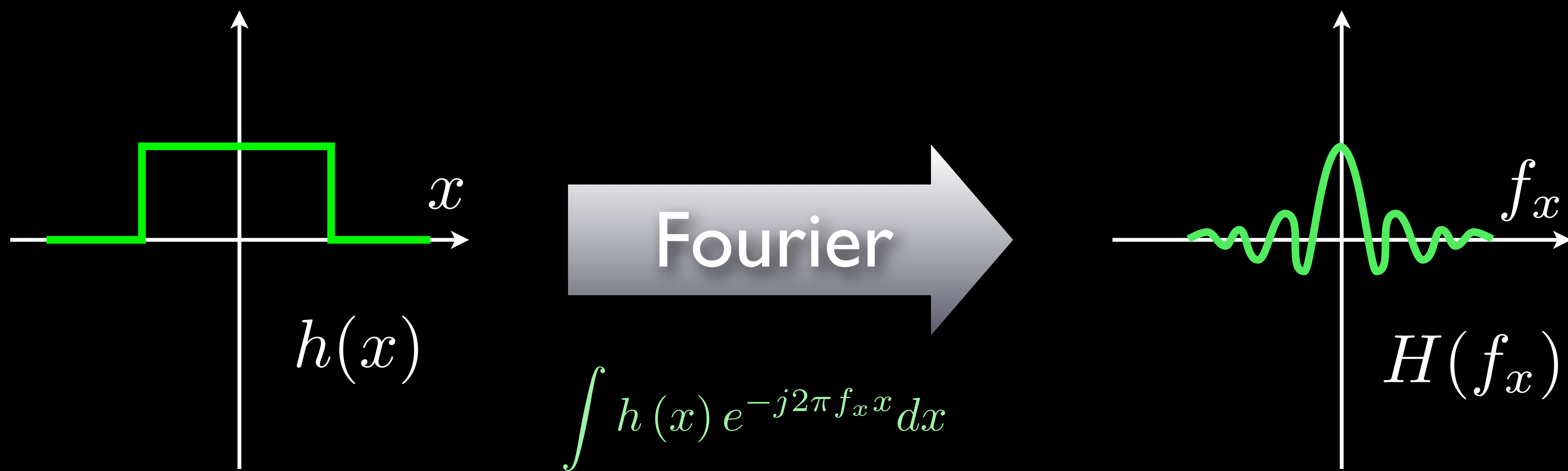

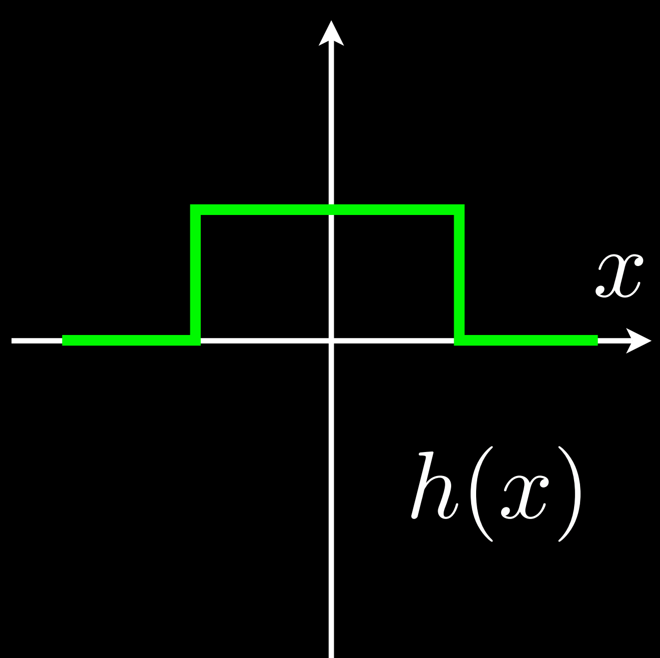

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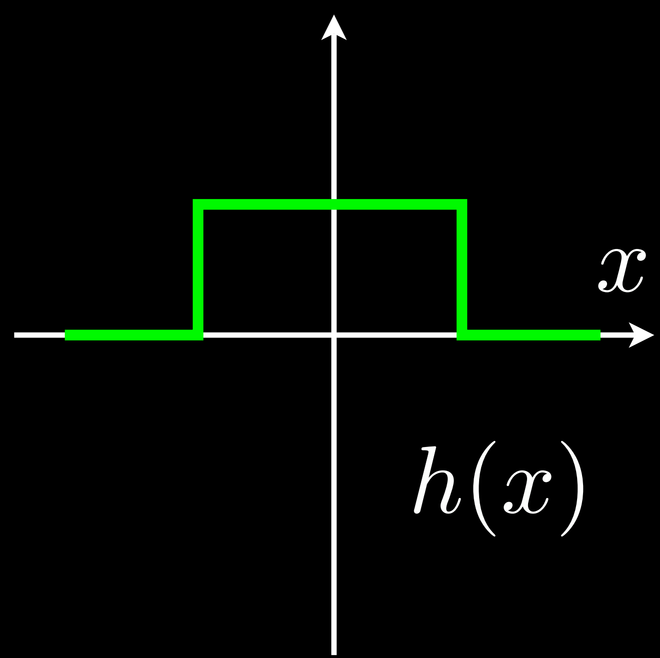
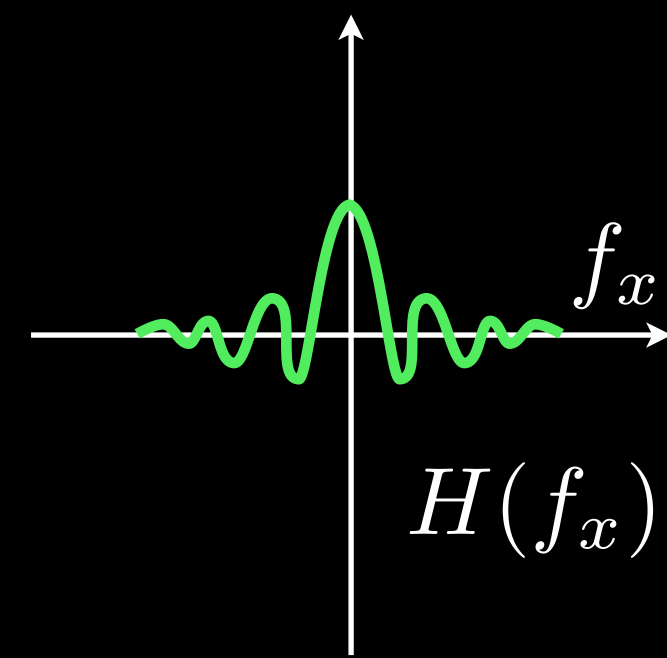
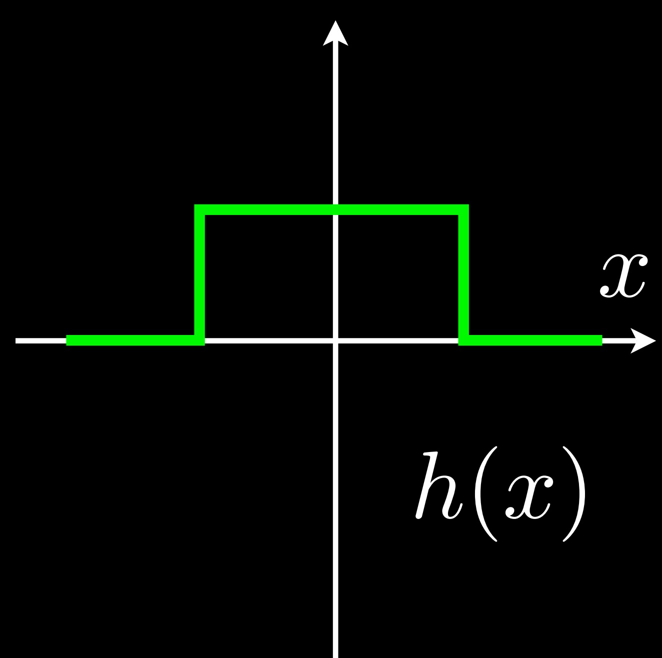


Fourier

$$\int h\left(x\right)e^{-j2\pi f_x x}dx$$

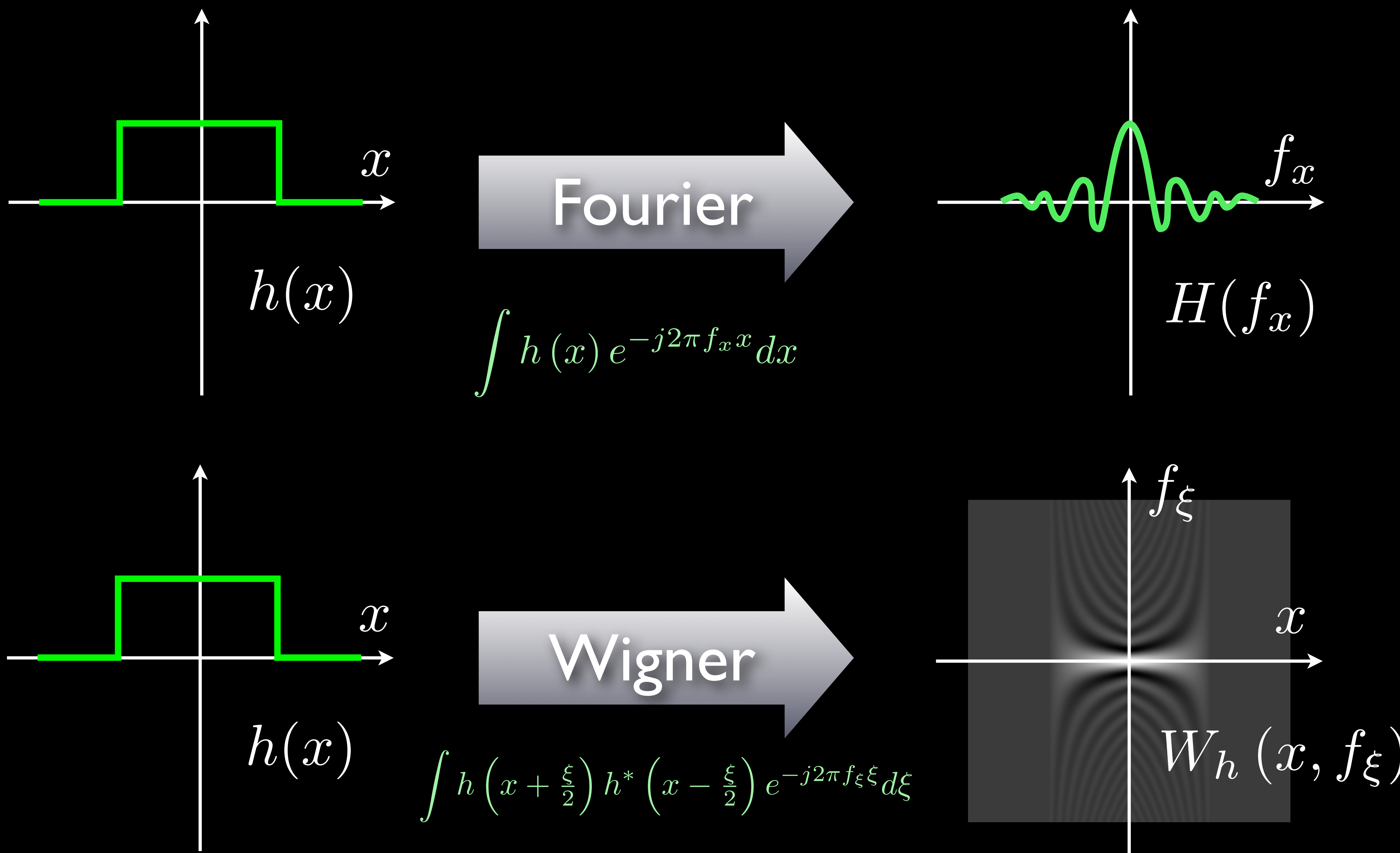




Wigner

$$\int h\left(x+\frac{\xi}{2}\right)h^*\left(x-\frac{\xi}{2}\right)e^{-j2\pi f_\xi \xi}d\xi$$

2D Wigner Distribution



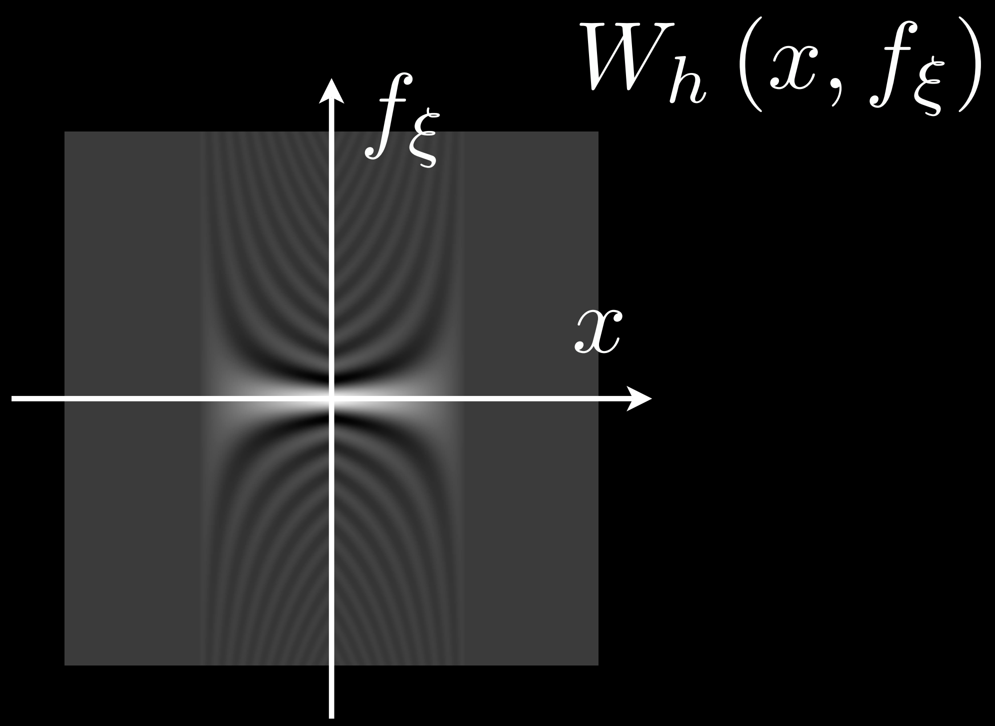
2D Wigner Distribution

$$W_h(x, f_\xi) = \int h\left(x + \frac{\xi}{2}\right) h^*\left(x - \frac{\xi}{2}\right) e^{-j2\pi f_\xi \xi} d\xi$$

- input: one-dimensional function of position
- output: two-dimensional function of position and frequency
- (some) information about spectrum at each position

2D Wigner Distribution

- projection along frequency yields power
- projection along position yields spectral power



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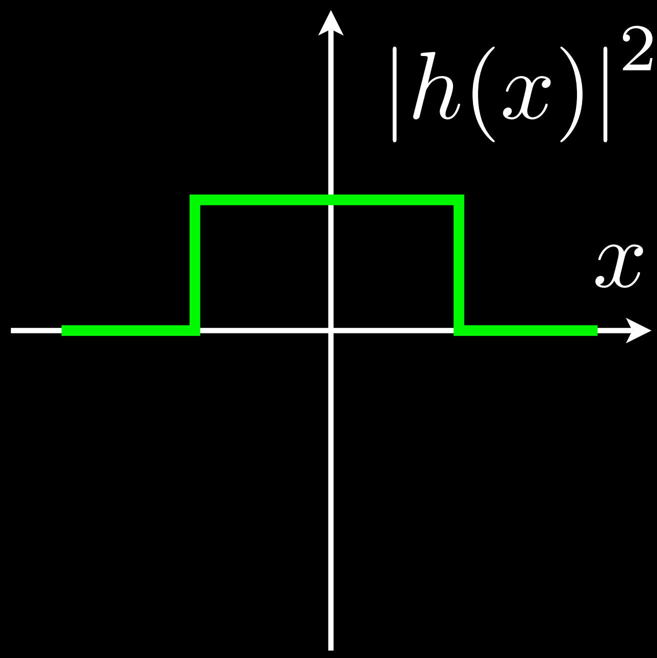

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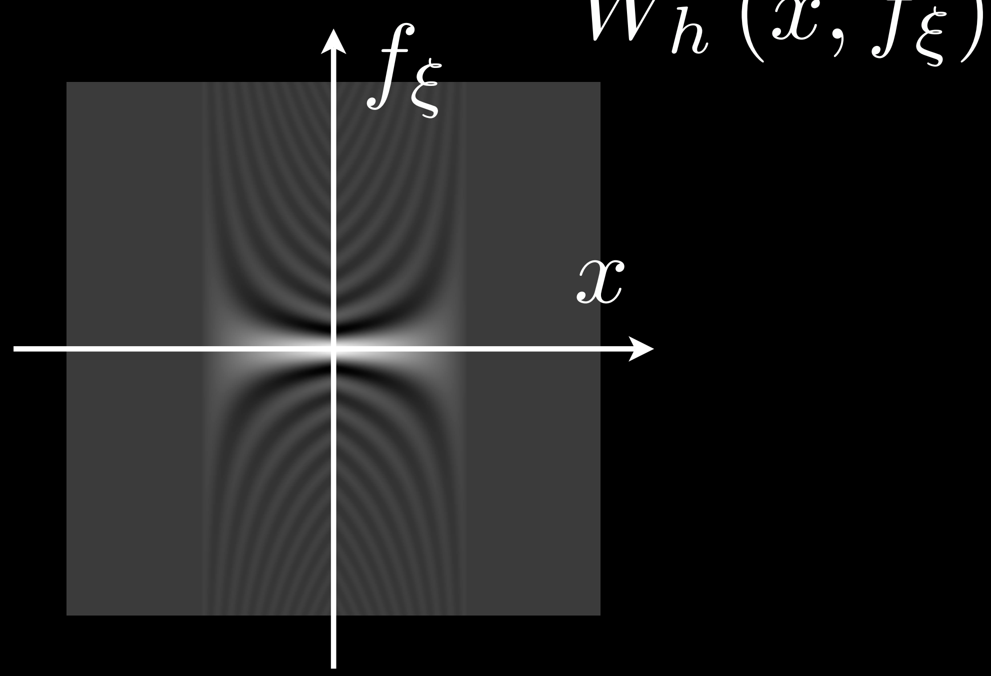

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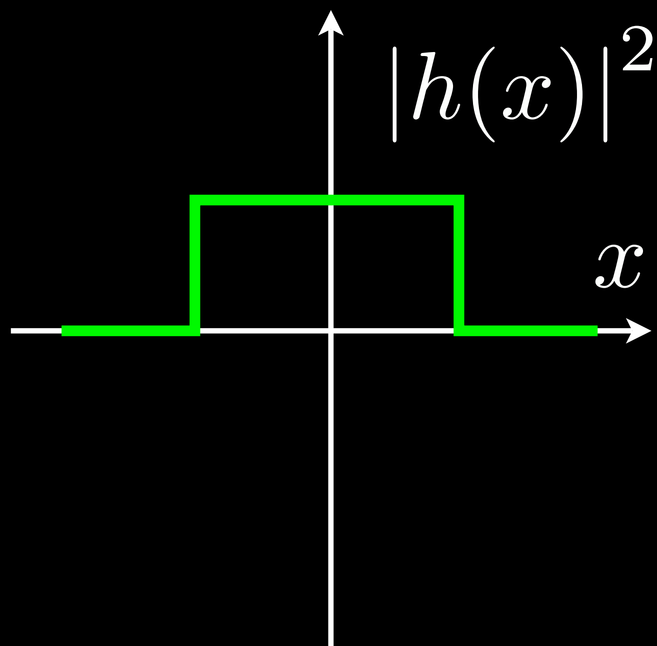

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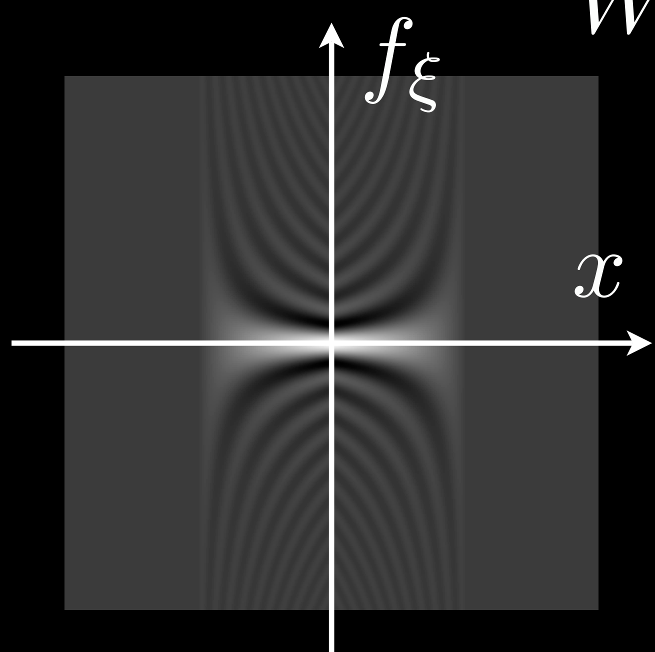
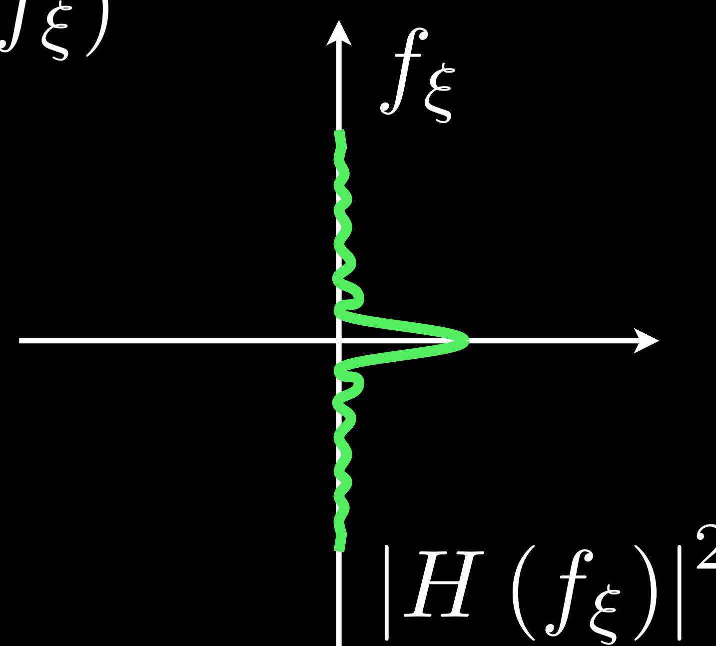
$W_h(x, f_\xi)$





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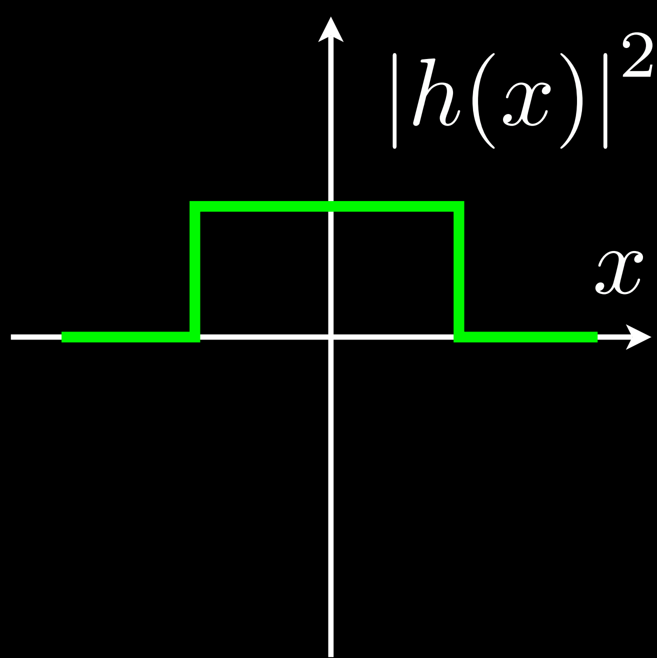
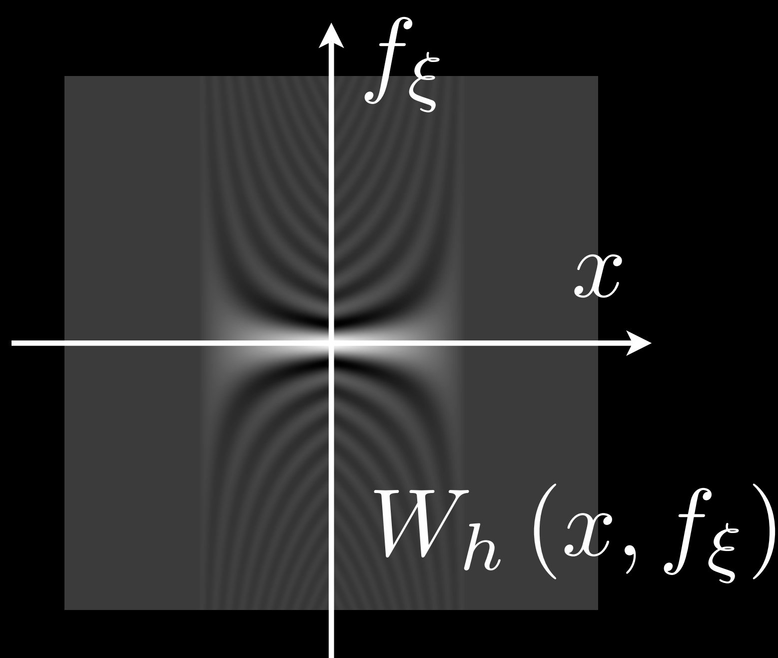
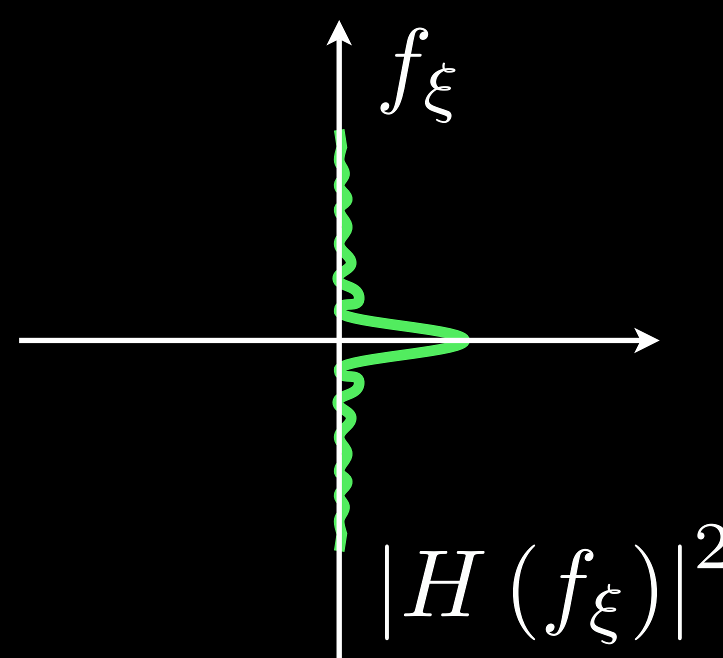


- tradeoff between width and height (fixed “area” or space-bandwidth product)
- uncertainty principle





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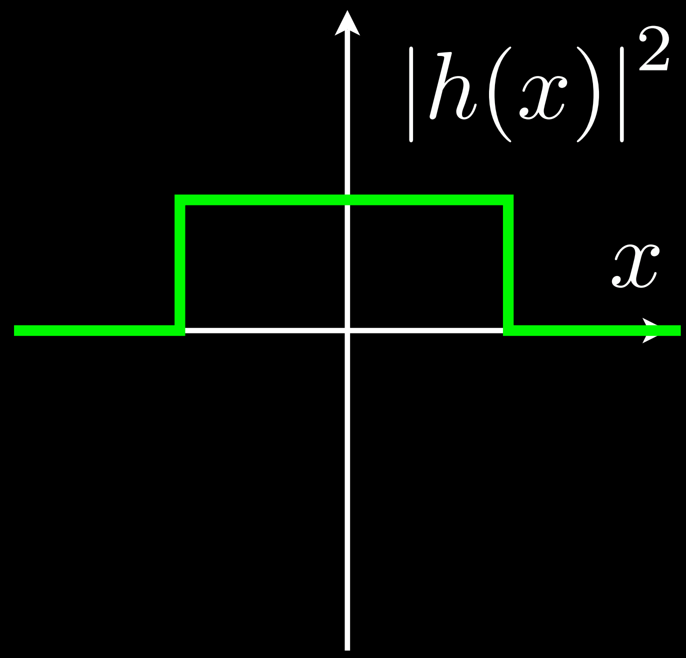

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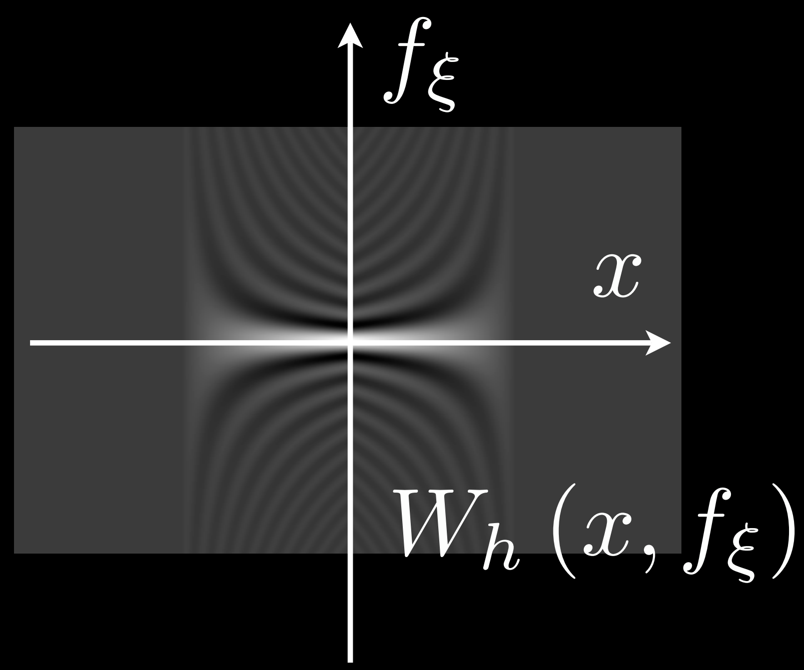

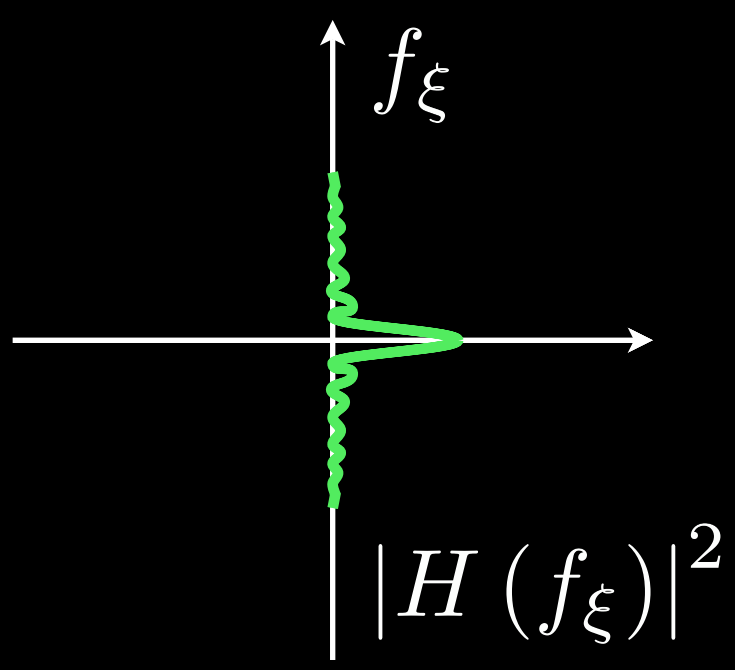

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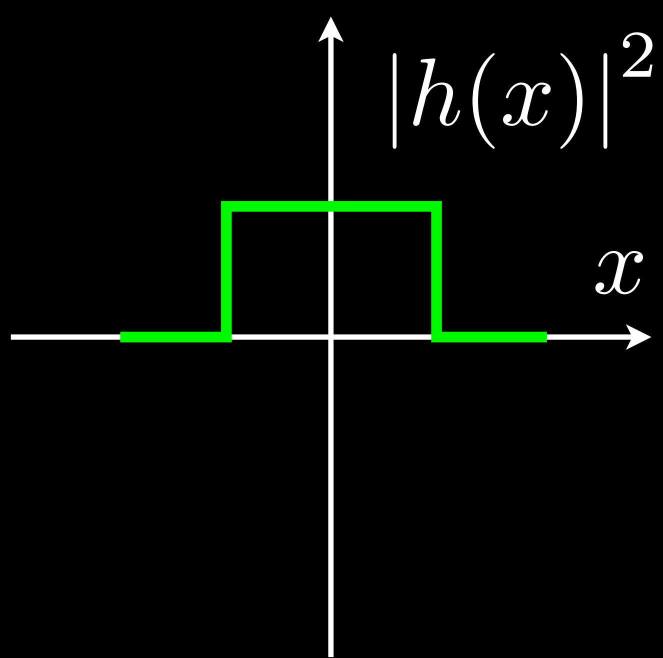

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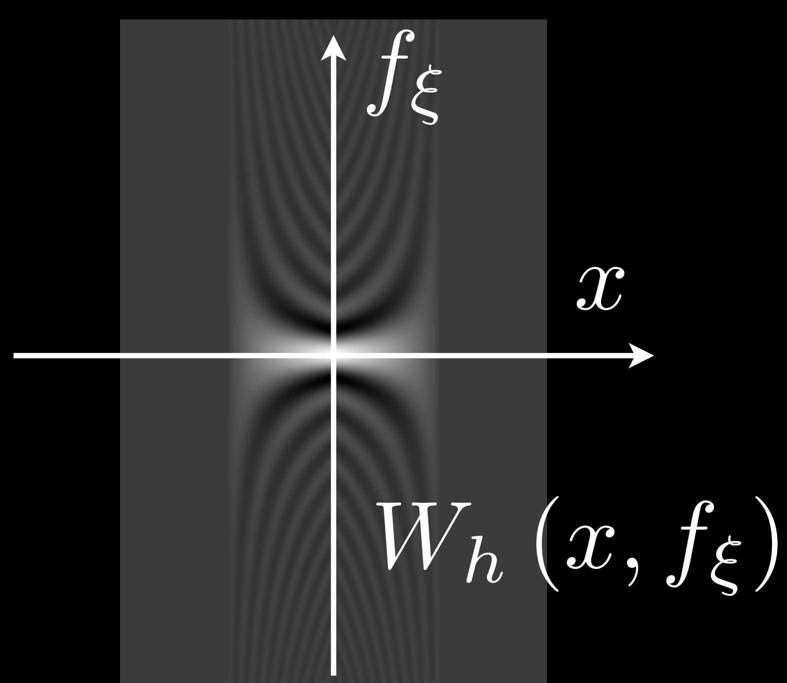

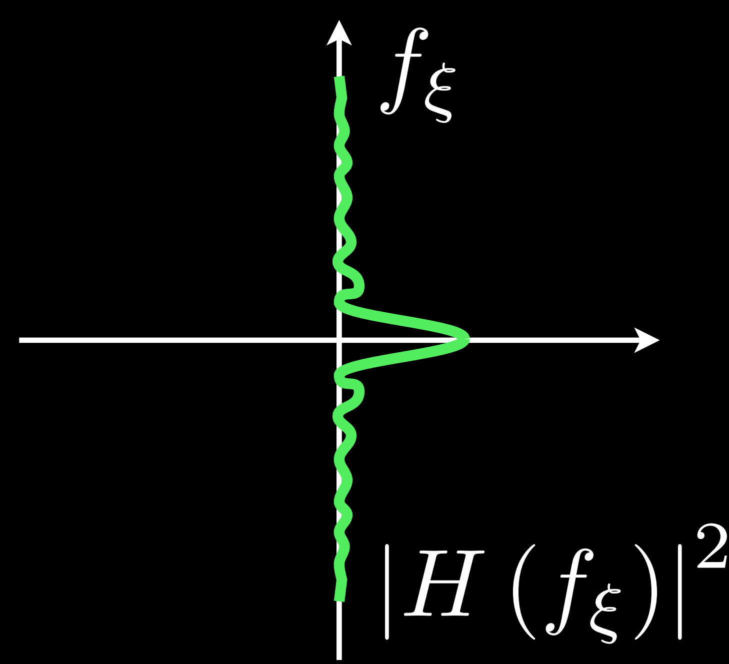

2D Wigner Distribution

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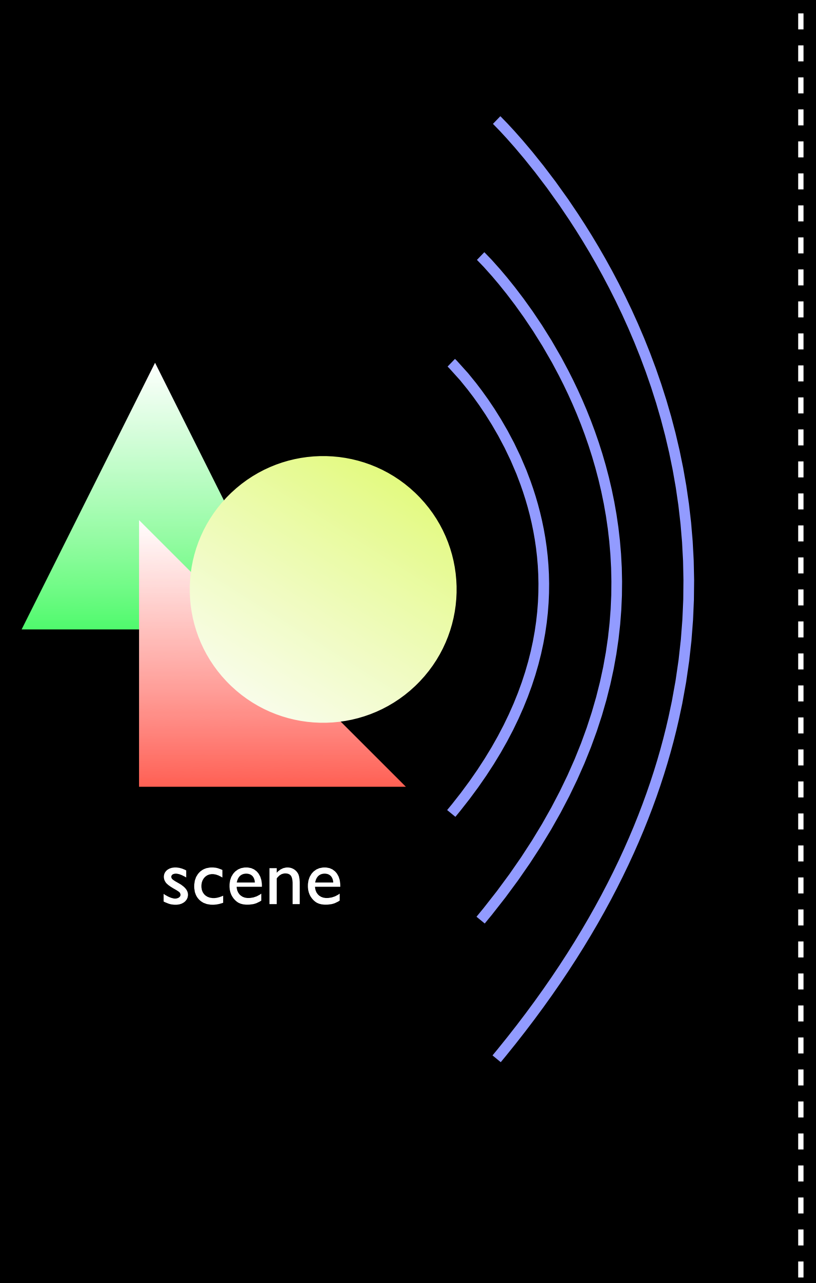
- information about both position and frequency
- fixed space-bandwidth product

Observable Light Field

- move aperture across plane
- look at directional spread
- continuous form of plenoptic camera



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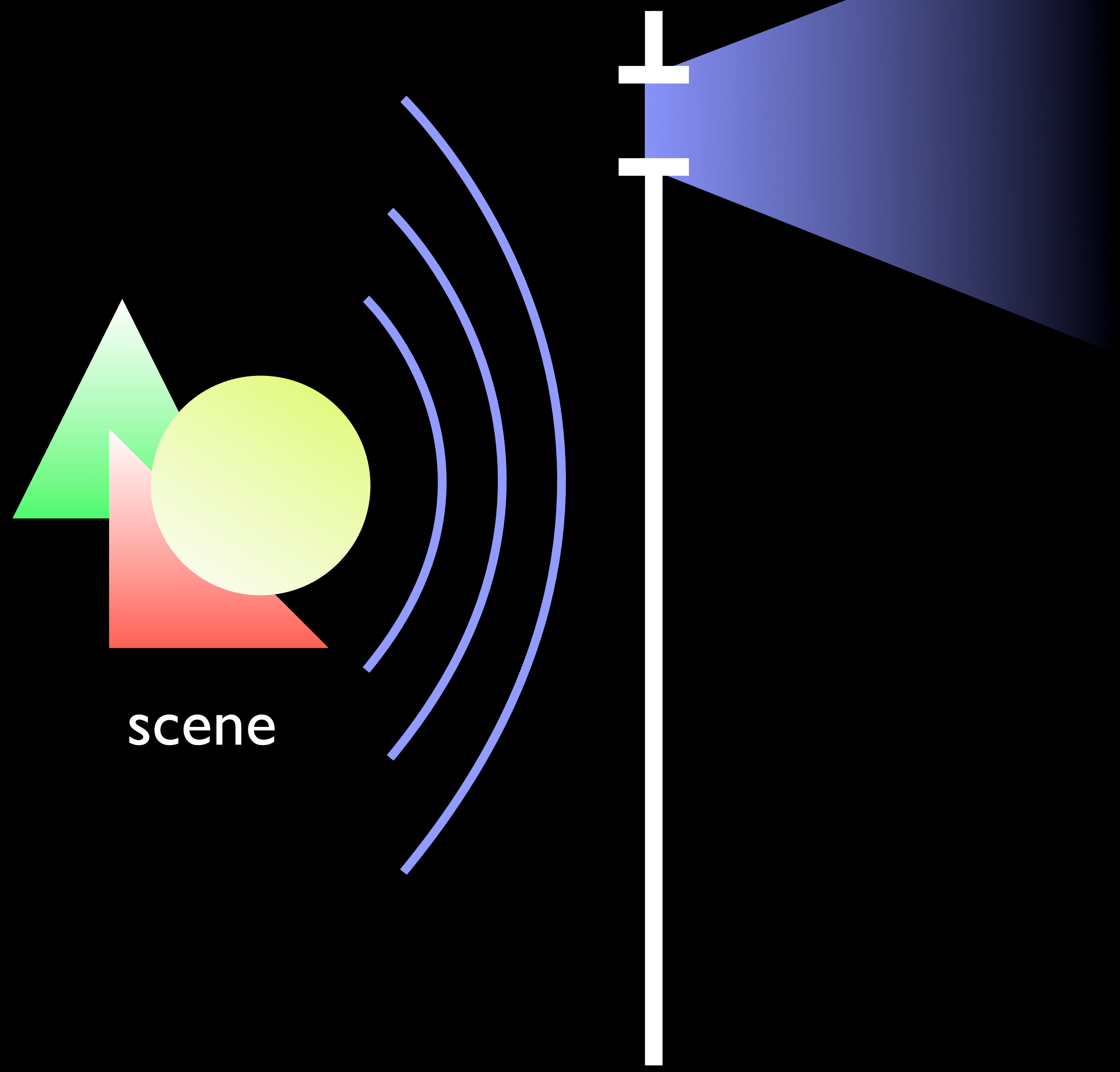

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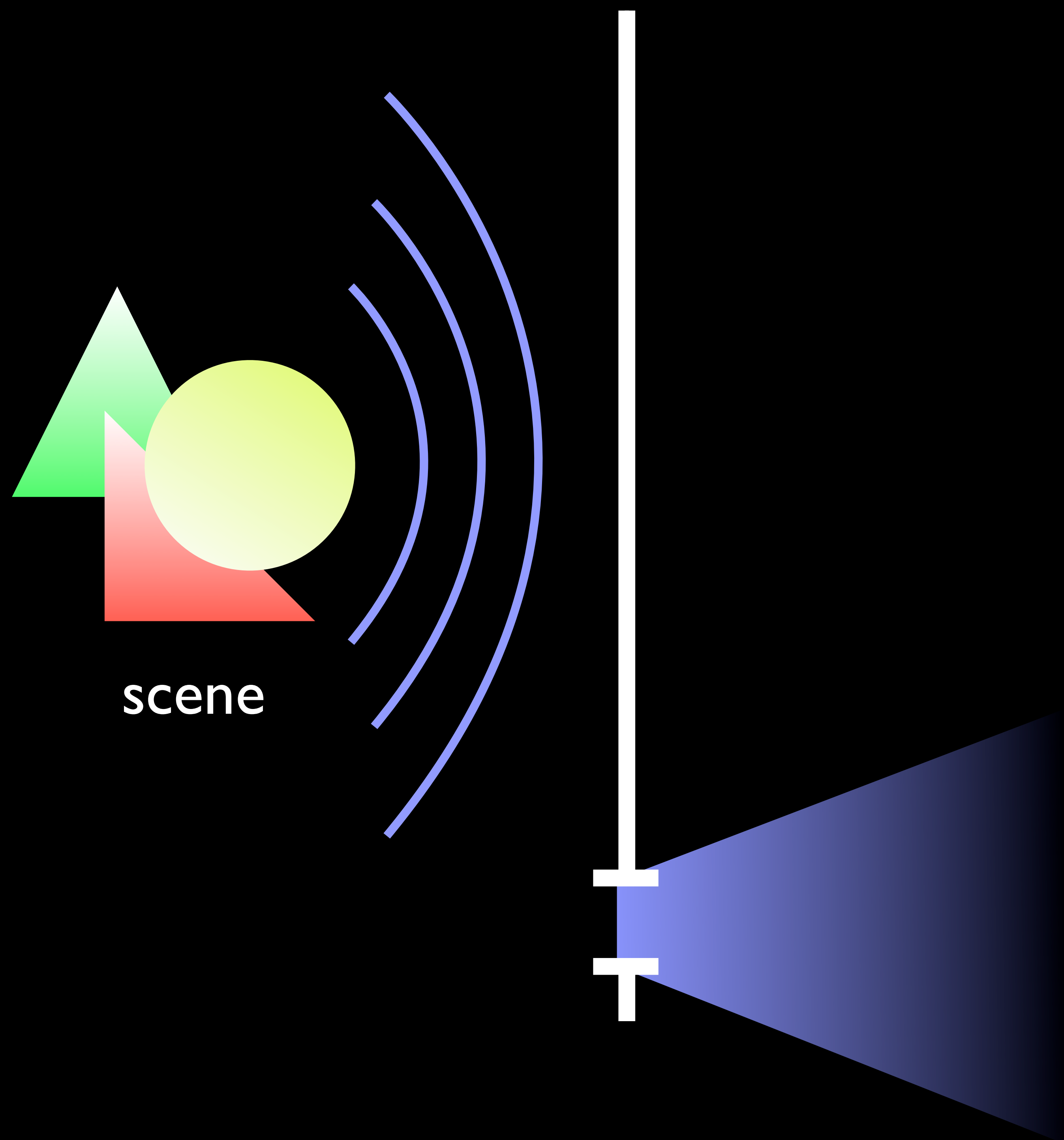

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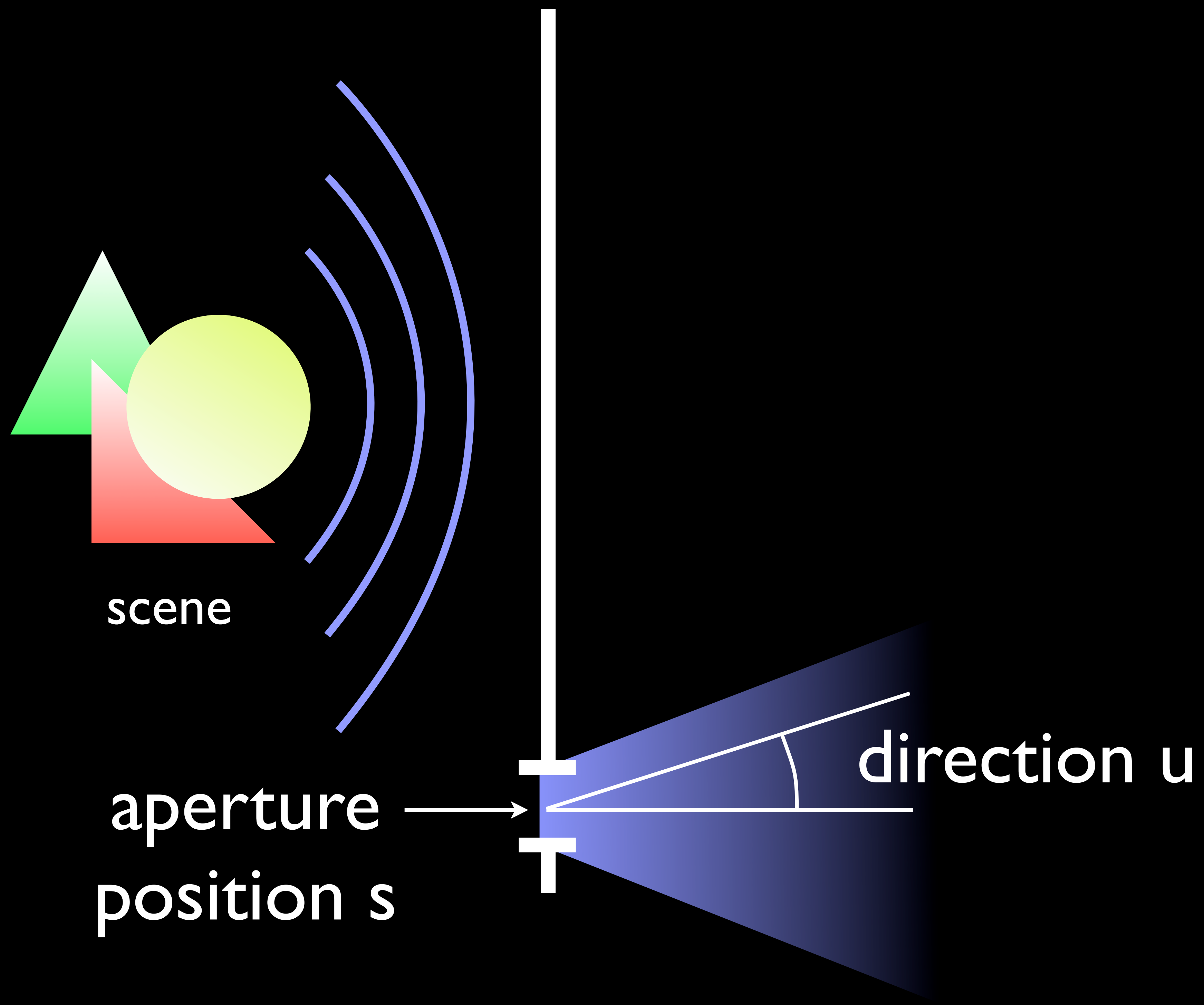
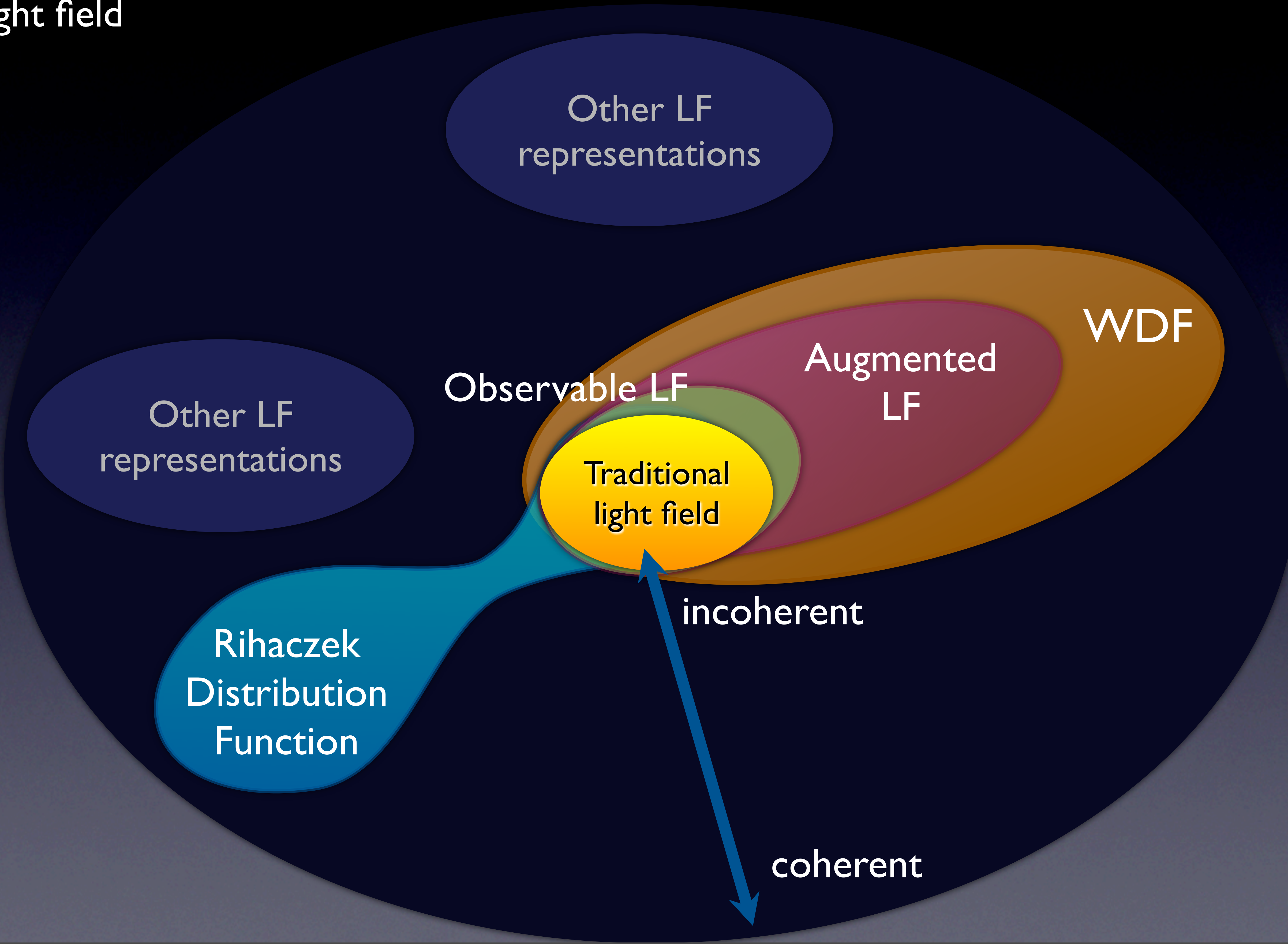


Space of LF representations
Time-frequency representations
Phase space representations
Quasi light field



Property of the Representation

	Constant along rays	Non-negativity	Coherence	Wavelength	Interference Cross term
Traditional LF	always constant	always positive	only incoherent	zero	no
Observable LF	nearly constant	always positive	any coherence state	any	yes
Augmented LF	only in the paraxial region	positive and negative	any	any	yes
WDF	only in the paraxial region	positive and negative	any	any	yes
Rihaczek DF	no; linear drift	complex	any	any	reduced

Benefits & Limitations of the Representation

	Ability to propagate	Modeling wave optics	Simplicity of computation	Adaptability to current pipe line	Near Field	Far Field
Traditional LF	x-shear	no	very simple	high	no	yes
Observable LF	not x-shear	yes	modest	low	yes	yes
Augmented LF	x-shear	yes	modest	high	no	yes
WDF	x-shear	yes	modest	low	yes	yes
Rihaczek DF	x-shear	yes	better than WDF, not as simple as LF	low	no	yes

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x) T(x-s) e^{-j2\pi \frac{u}{\lambda} x} dx \right|^2$$

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi\frac{u}{\lambda}x}dx \right|^2$$

Fourier transform

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x) T(x-s) e^{-j2\pi \frac{u}{\lambda} x} dx \right|^2$$

wave

Fourier transform

Observable Light Field

aperture window

$$l_{obs}^{(T)}(s,u) = \left| \int U(x) T(x - s) e^{-j2\pi \frac{u}{\lambda} x} dx \right|^2$$

wave

Fourier transform

Observable Light Field

aperture window

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi \frac{u}{\lambda} x} dx \right|^2$$

power

wave

Fourier transform

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi\frac{u}{\lambda}x}dx \right|^2$$

$$\Downarrow$$

$$l_{obs}^{(T)}(s,u) = W_U\left(s,\frac{u}{\lambda}\right) \otimes W_T\left(-s,\frac{u}{\lambda}\right)$$

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi\frac{u}{\lambda}x}dx \right|^2$$

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$$l_{obs}^{(T)}(s,u) = W_U\left(s,\frac{u}{\lambda}\right) \otimes W_T\left(-s,\frac{u}{\lambda}\right)$$

Wigner distribution of wave function

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi \frac{u}{\lambda} x} dx \right|^2$$

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Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

$$l^{(T)}_{obs}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi \frac{u}{\lambda} x}dx \right|^2$$

blur trades off resolution in position with direction

$$l^{(T)}_{obs}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes W_T\left(-s, \frac{u}{\lambda}\right)$$

Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes W_T\left(-s, \frac{u}{\lambda}\right)$$

Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes \delta(-s,u)$$

Wigner distribution
of wave function

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s, u) = W_U\left(s, \frac{u}{\lambda}\right)$$

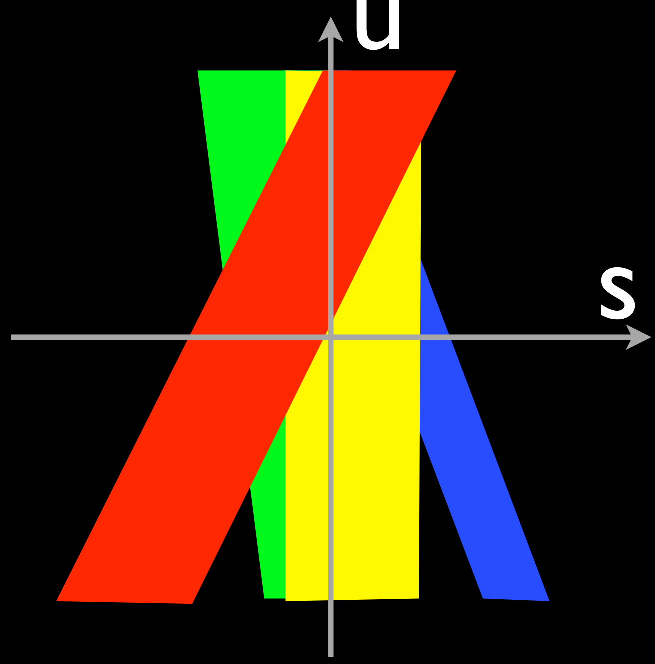
observable light field and Wigner equivalent!

Observable Light Field

- observable light field is a blurred Wigner distribution with a modified coordinate system
- blur trades off resolution in position with direction
- Wigner distribution and observable light field equivalent at zero wavelength limit

Application - Refocusing

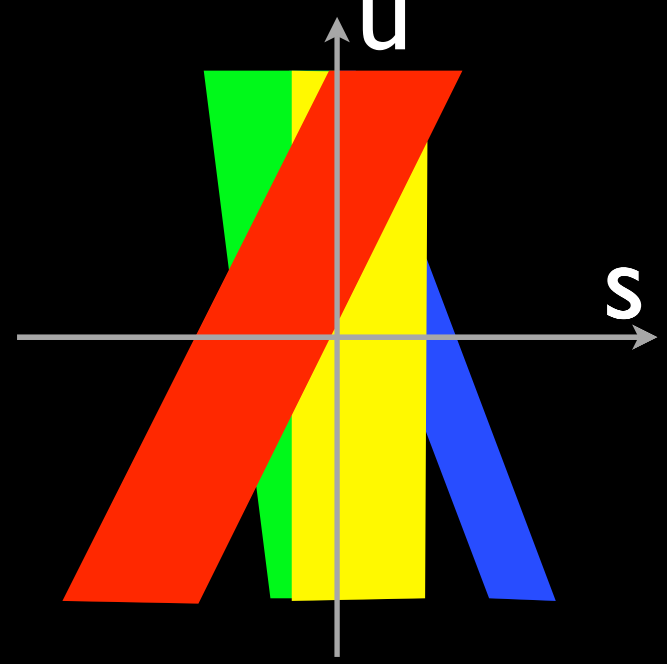
light
field



Application - Refocusing

Isaksen et. al 2000

light field



Application - Refocusing

Isaksen
et. al
2000

light
field

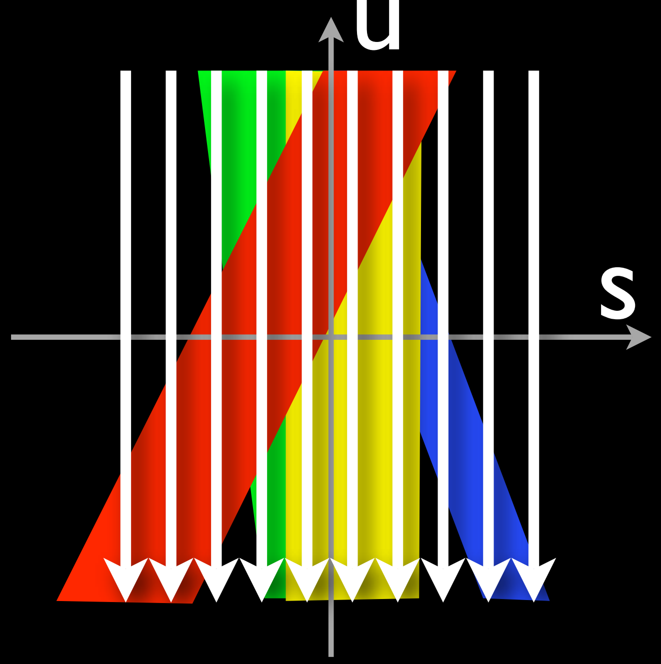


image at z=0

Application - Refocusing

Isaksen
et. al
2000

light
field

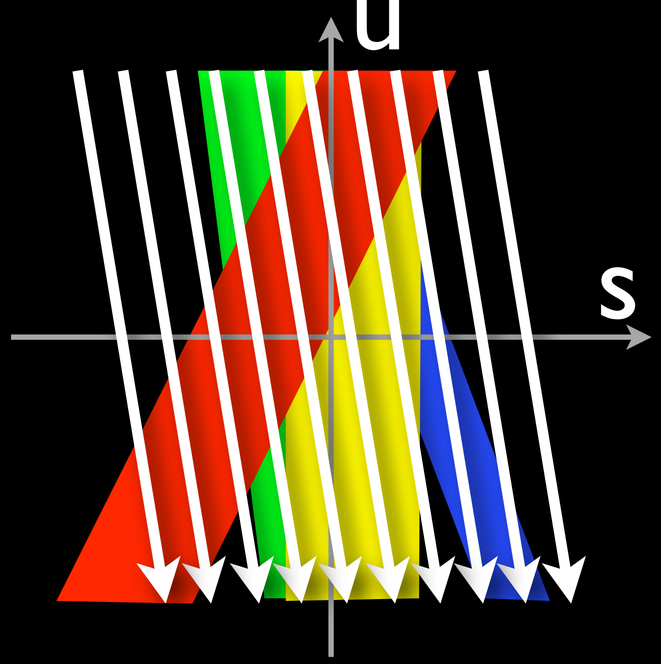
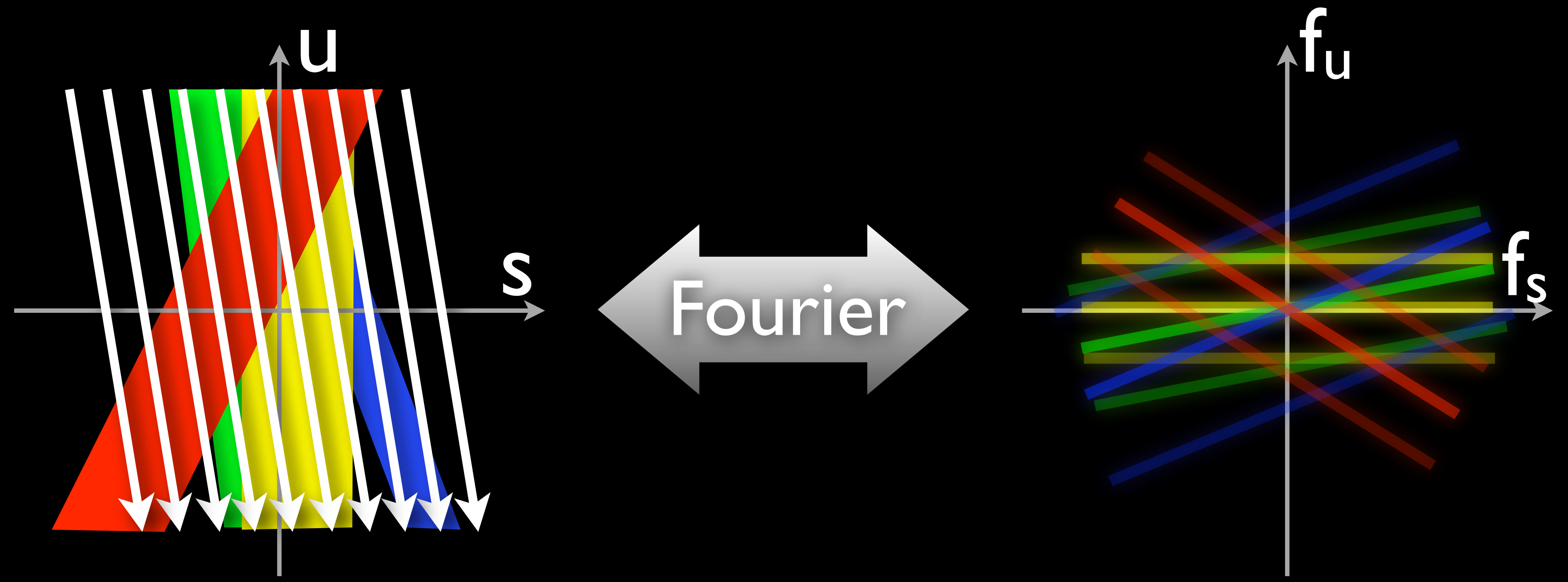


image at z=z_0

Application - Refocusing

Isaksen et. al 2000

light field

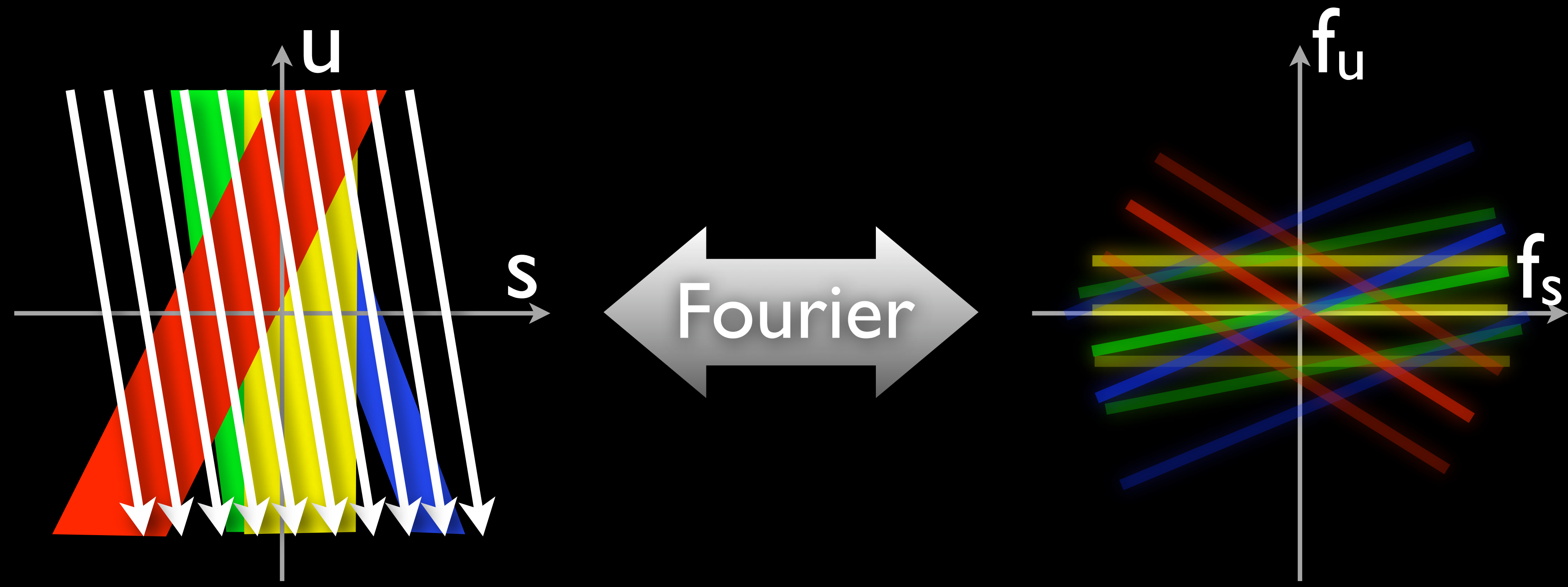


light field spectrum

Application - Refocusing

Isaksen et. al 2000

light field



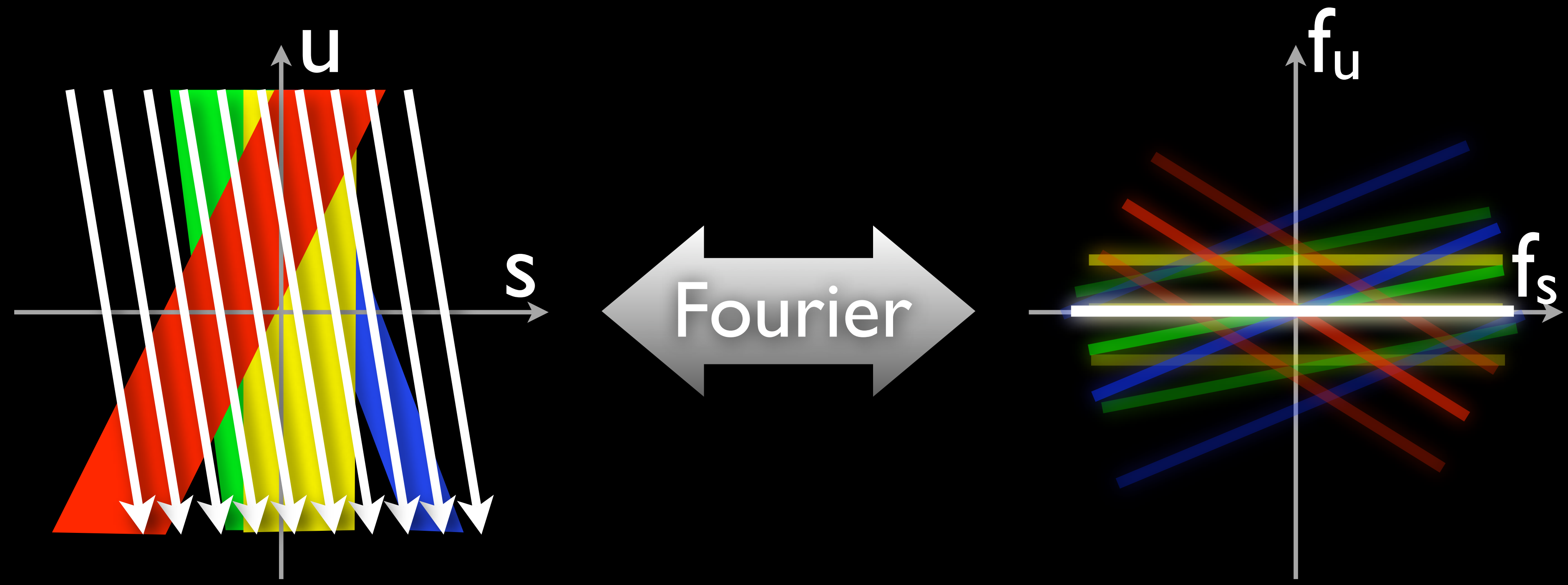
Ng 2005

light field spectrum

Application - Refocusing

Isaksen
et. al
2000

light
field



Ng
2005

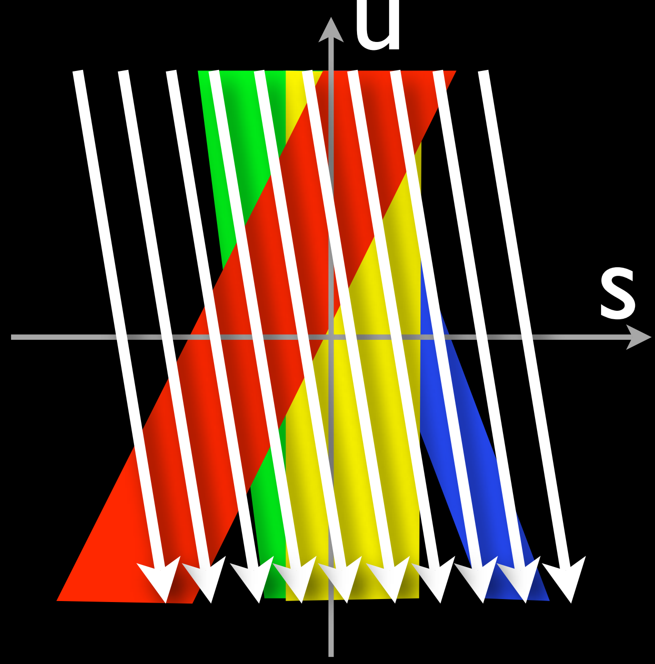
light
field
spectrum

image at z=0

Application - Refocusing

Isaksen et. al 2000

light field



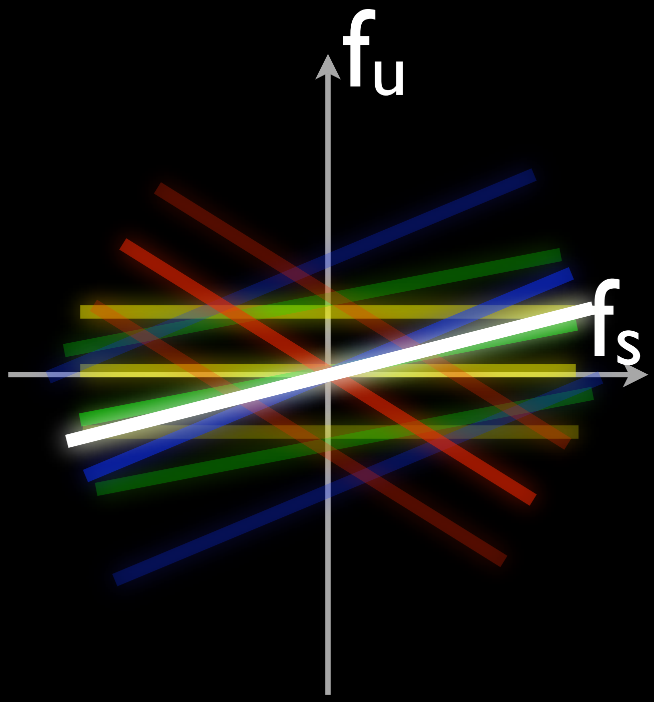
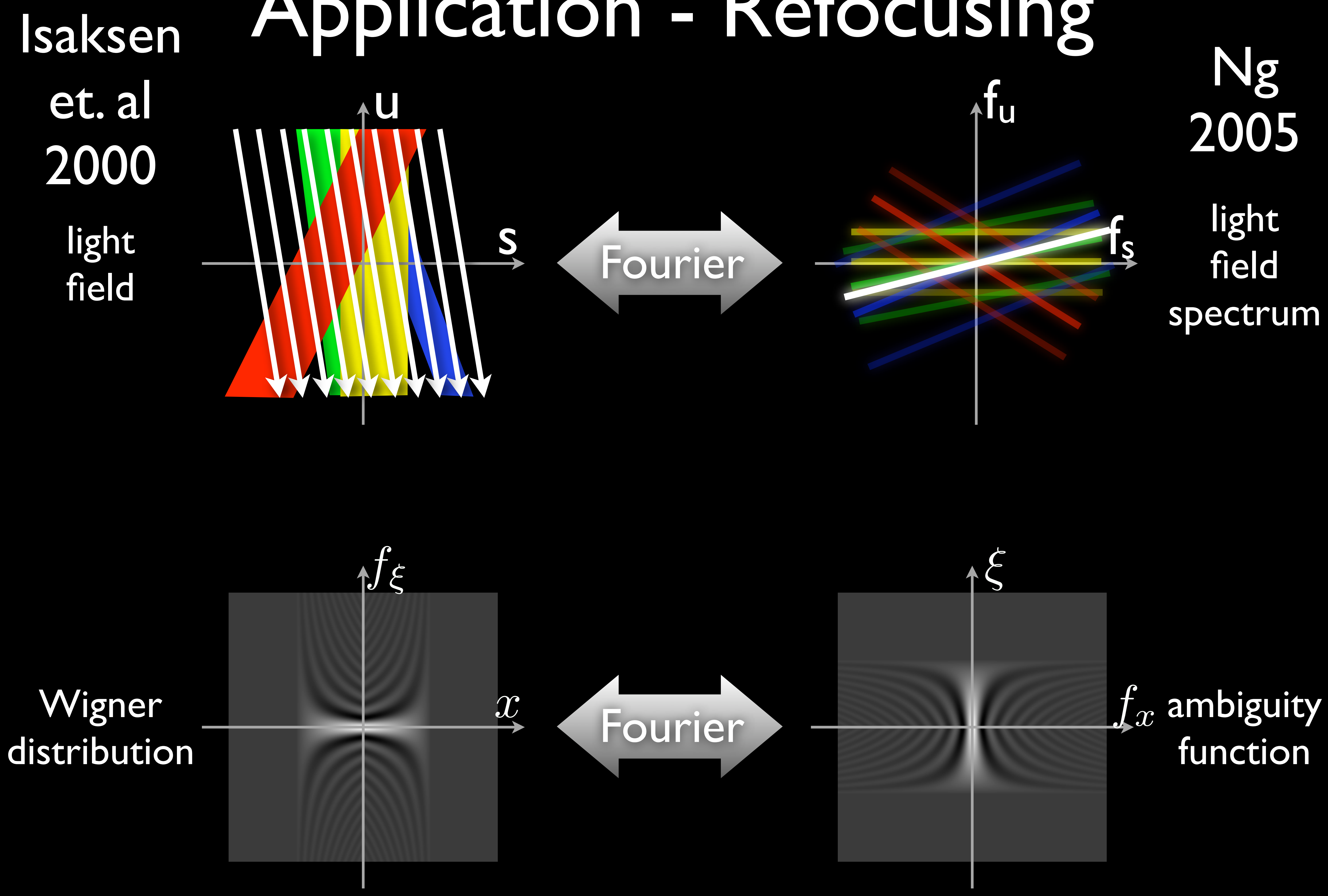


image at $z=z_0$

Ng 2005

light field spectrum

Application - Refocusing



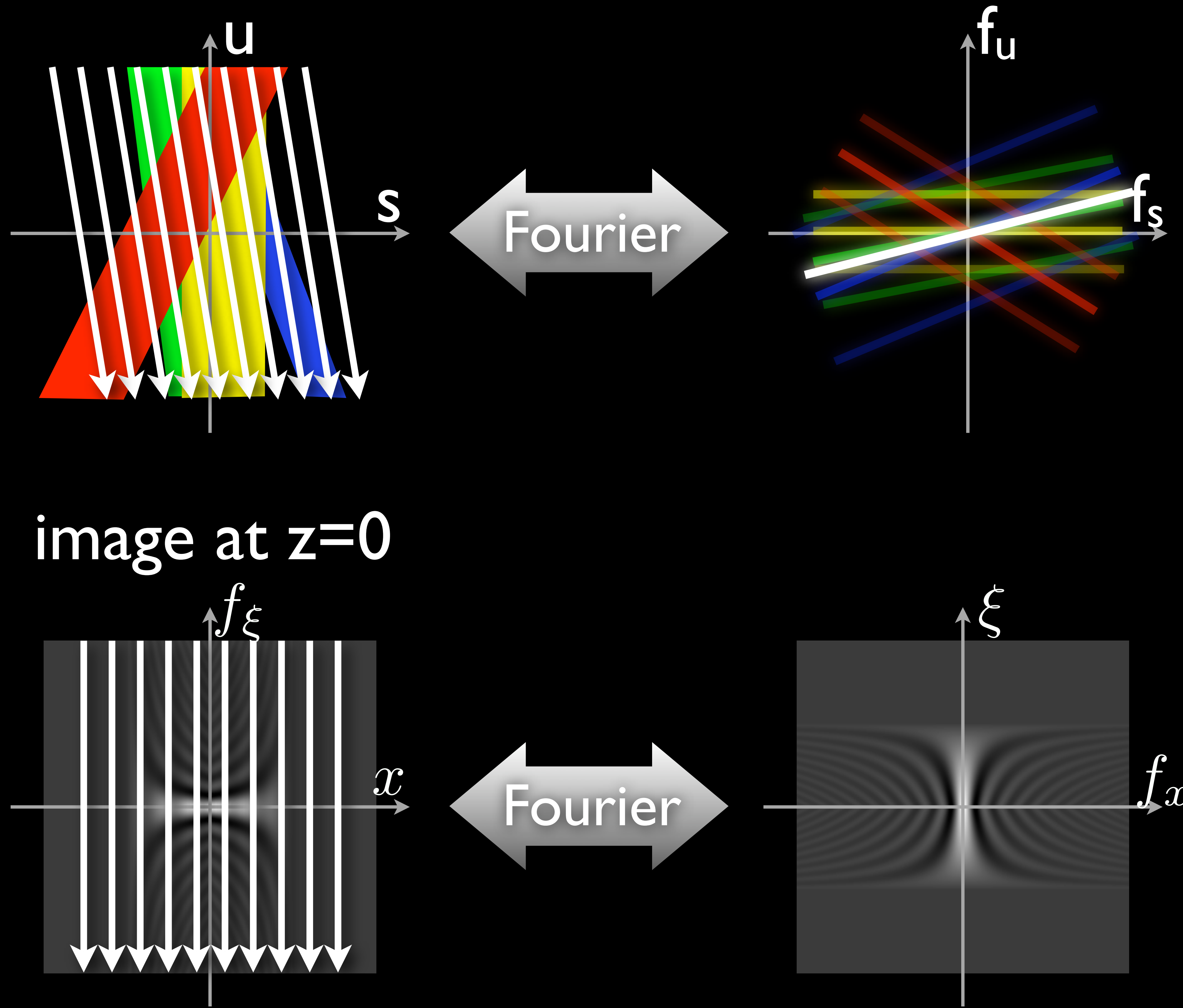
Application - Refocusing

Isaksen
et. al
2000

light
field

Ng
2005

light
field
spectrum



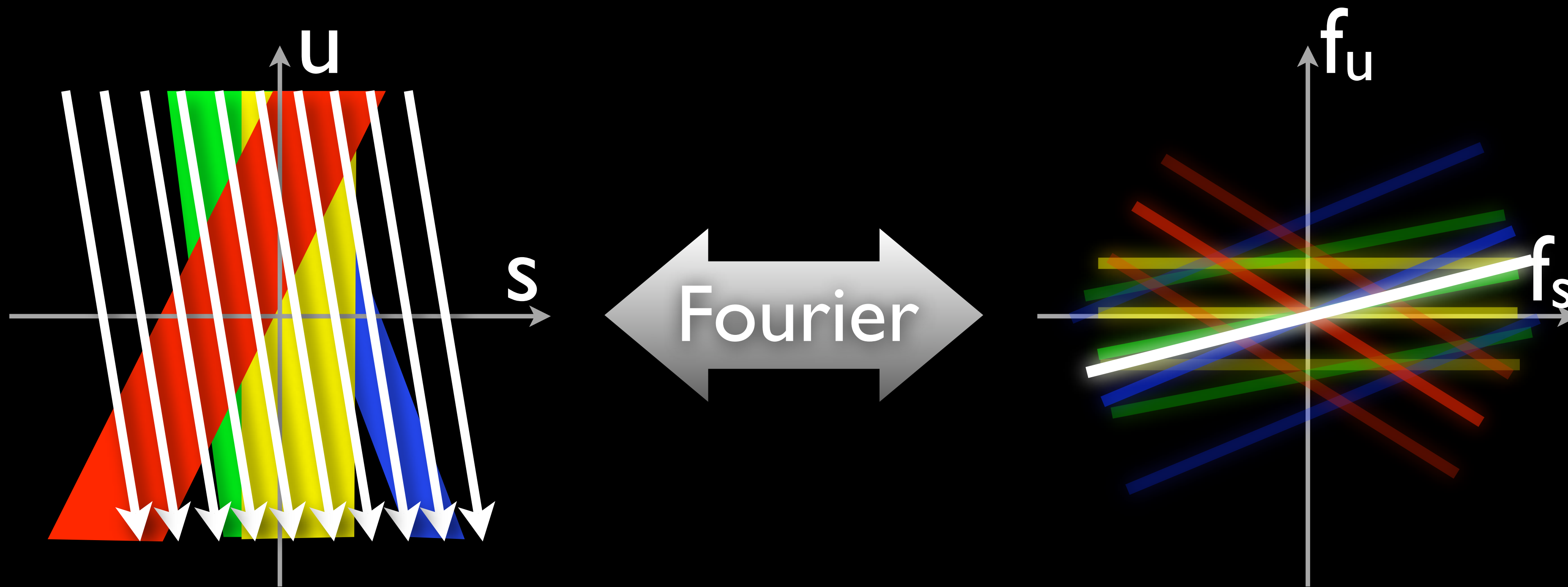
Wigner
distribution

ambiguity
function

Application - Refocusing

Isaksen et. al 2000

light field

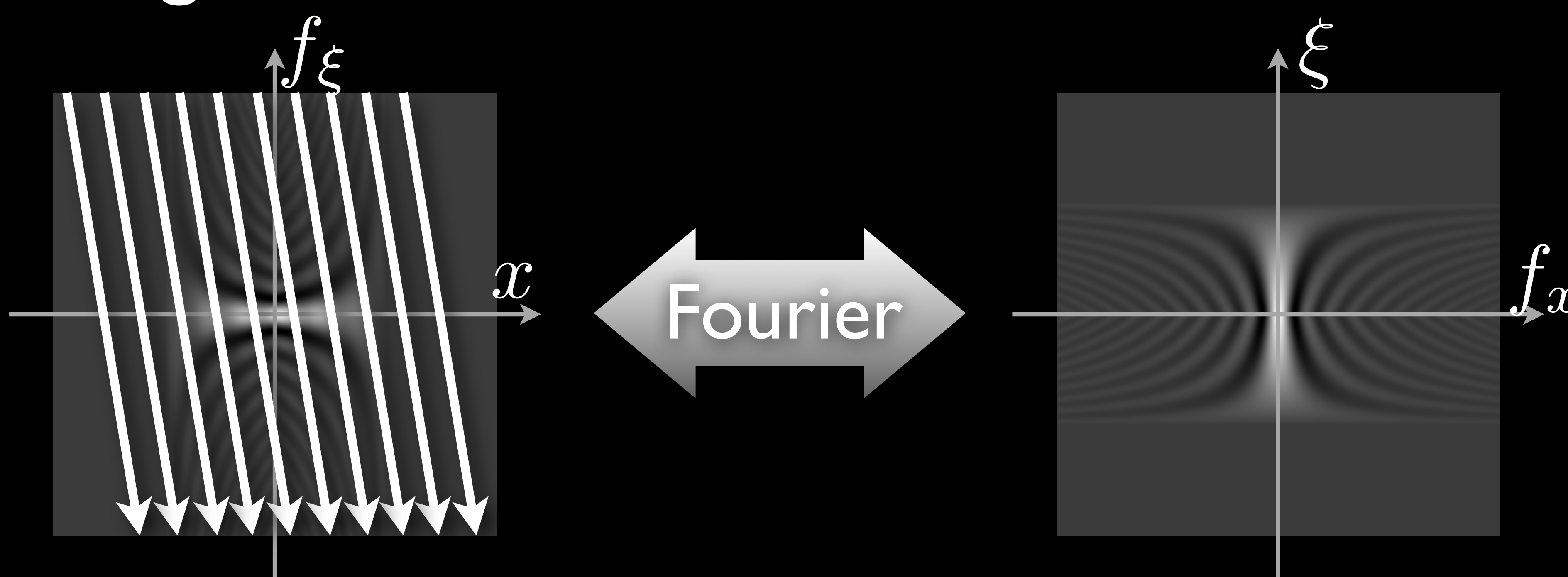


Ng 2005

light field spectrum

image at $z=z_0$

Wigner distribution

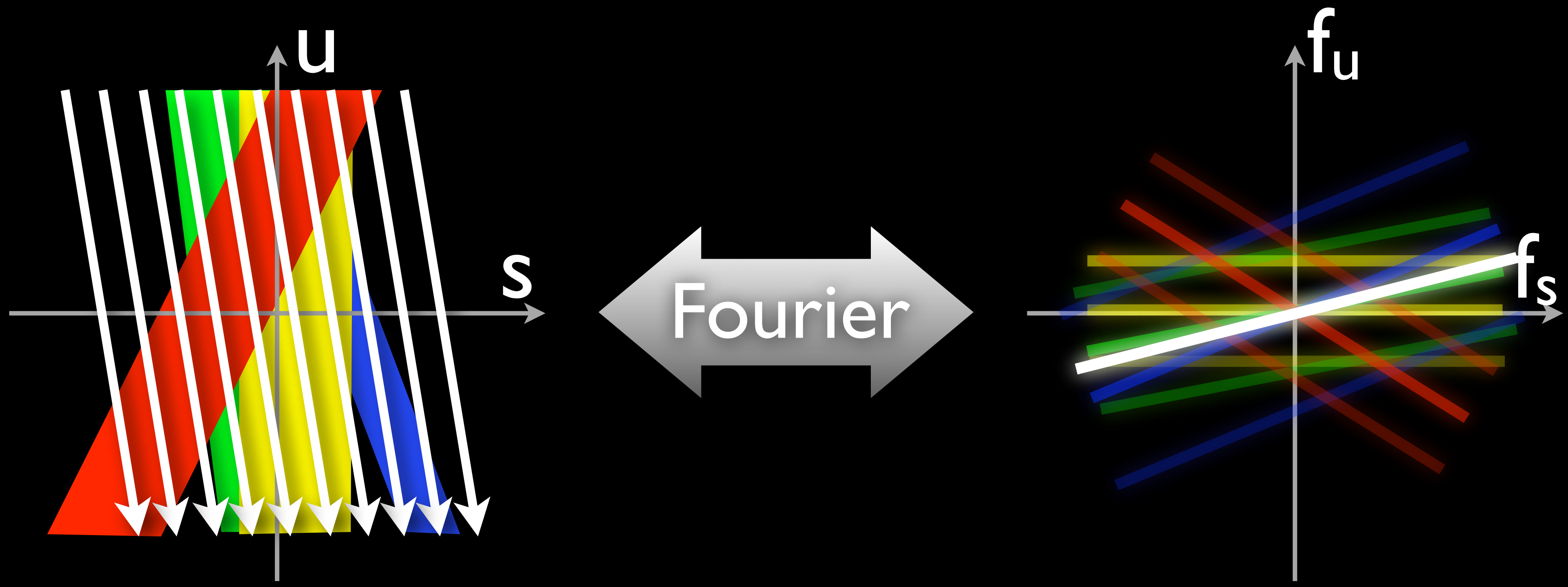


ambiguity function

Application - Refocusing

Isaksen et. al 2000

light field

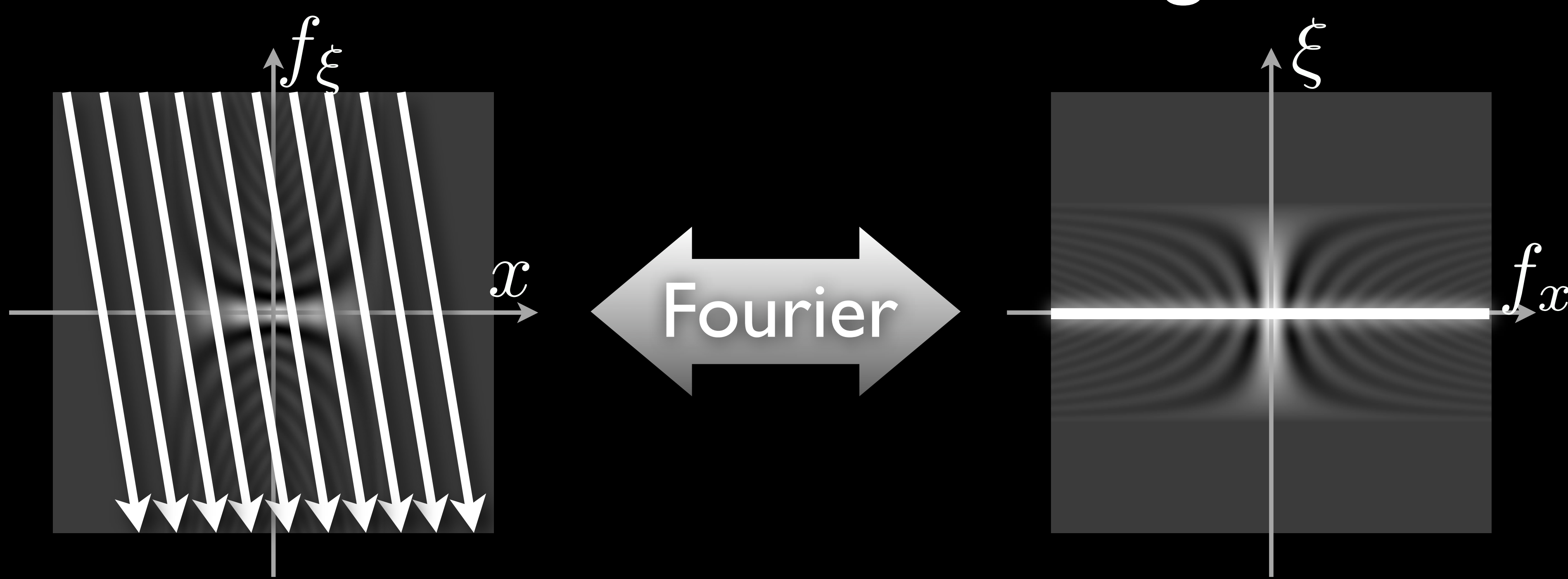


Ng 2005

light field spectrum

image at z=0

Wigner distribution

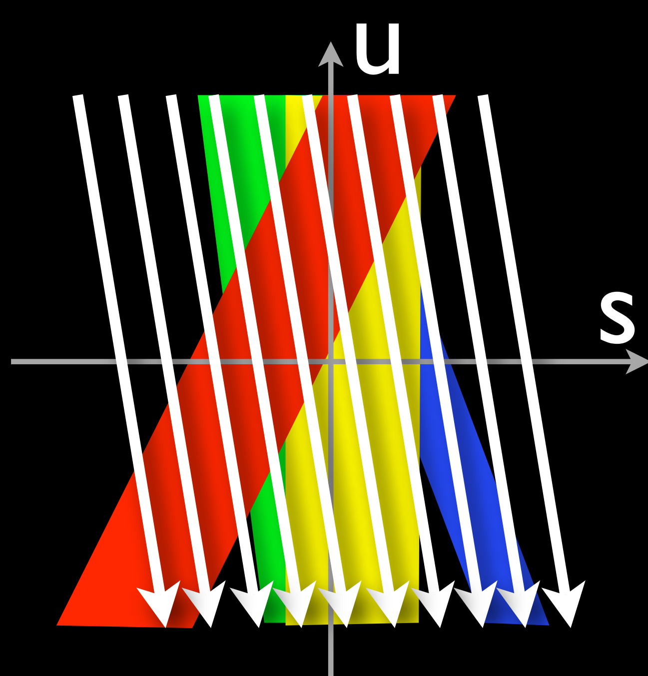


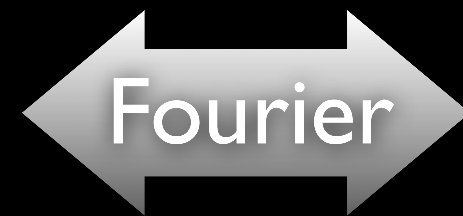
ambiguity function

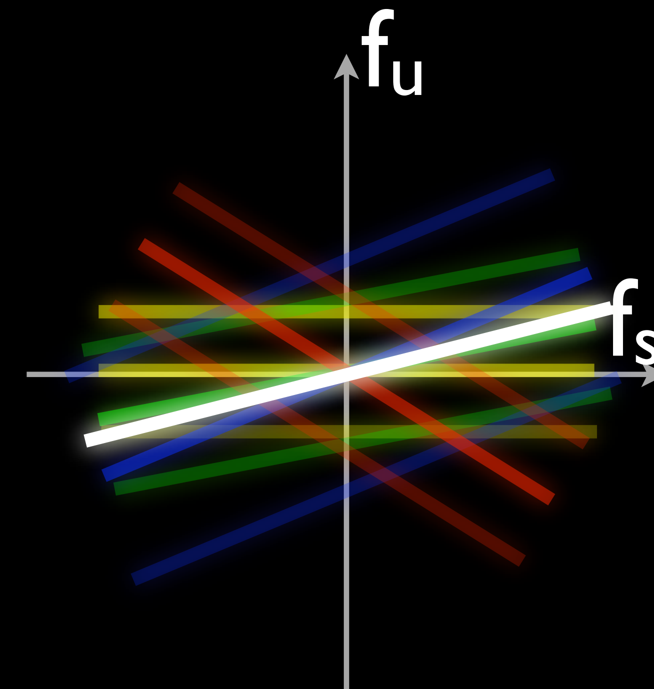
Application - Refocusing

Isaksen et. al 2000

light field





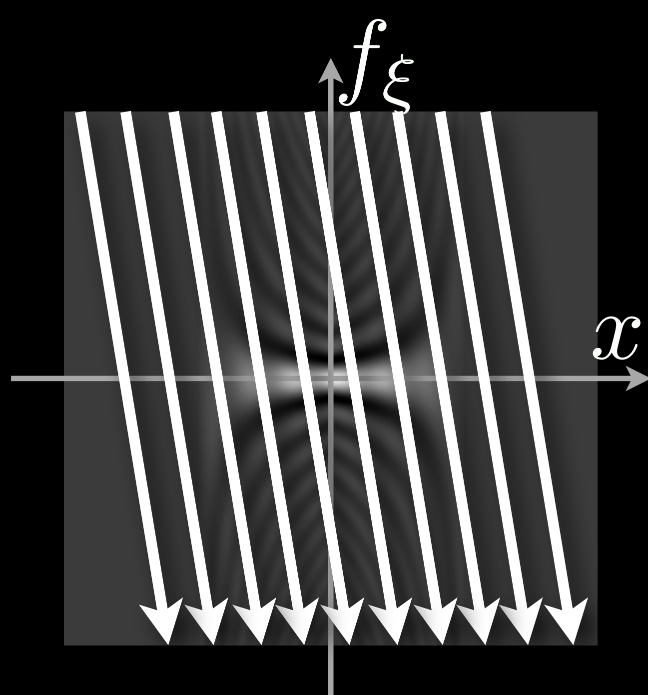


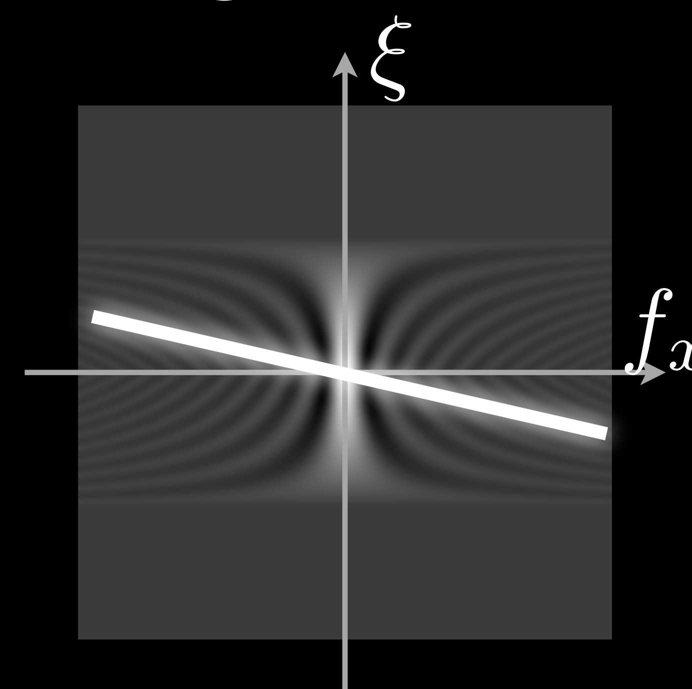
Ng 2005

light field spectrum

image at $z=z_0$

Wigner distribution



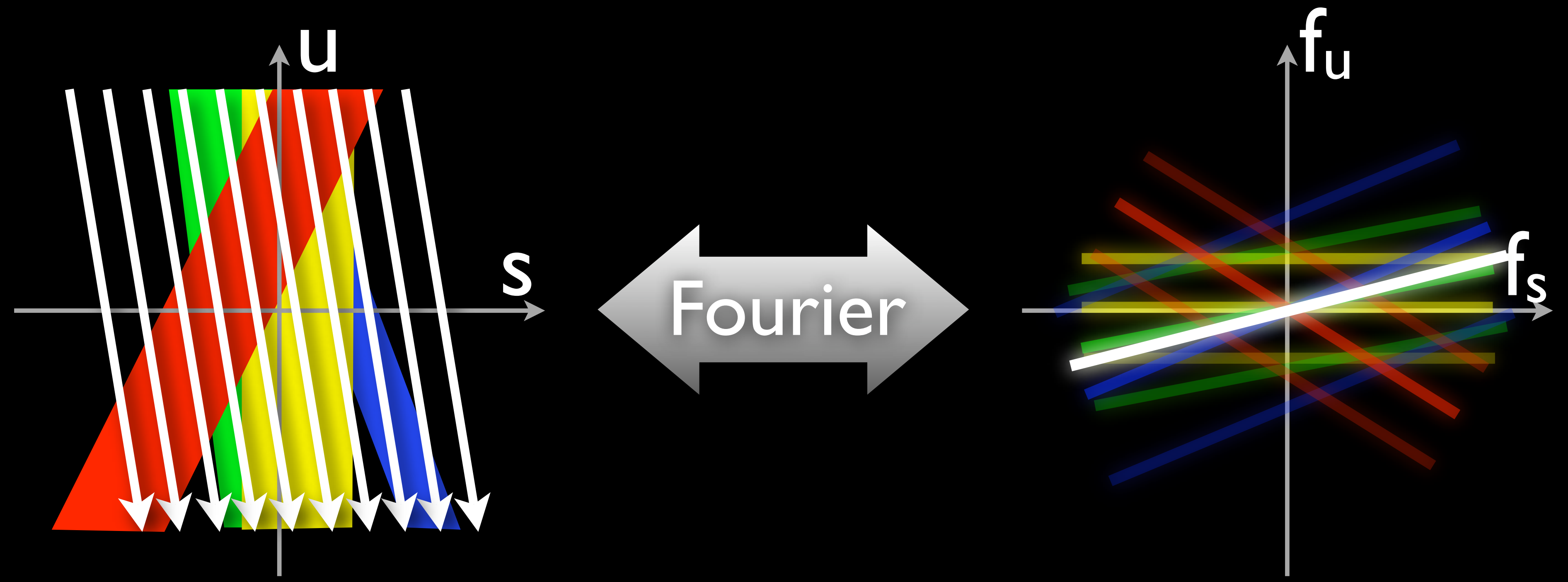


ambiguity function

Application - Refocusing

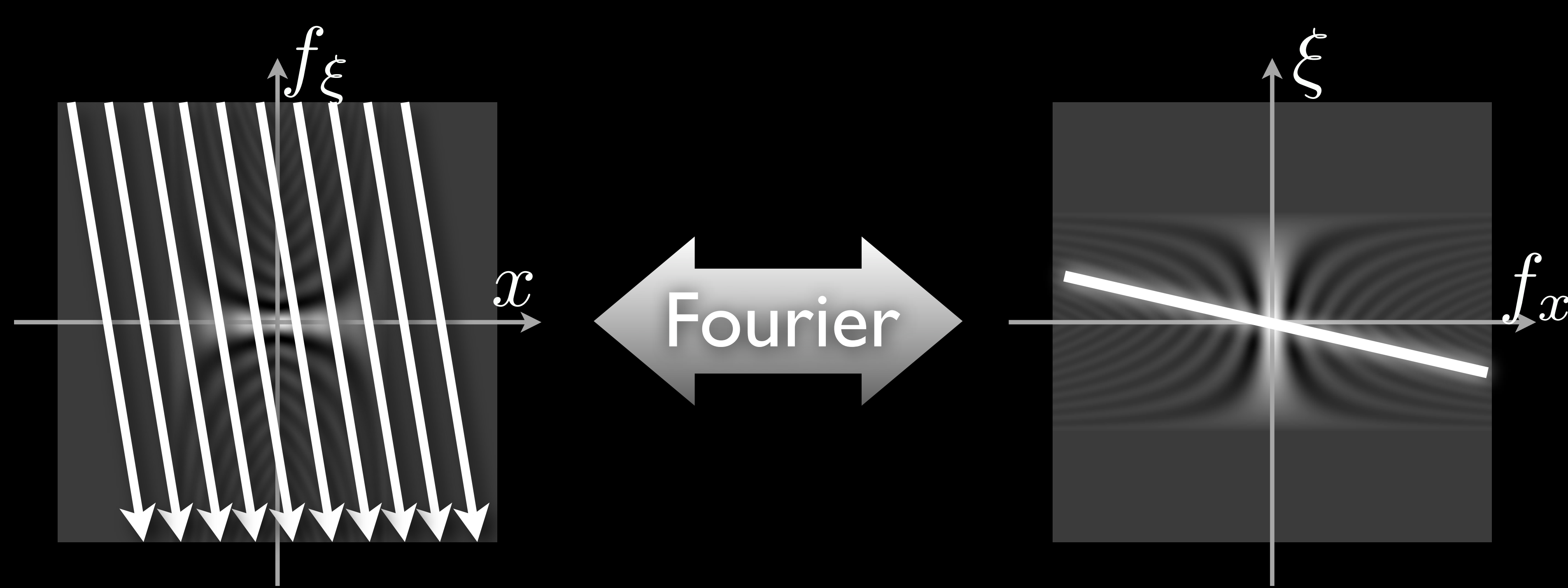
Isaksen et. al 2000

light field



Ng 2005

light field spectrum



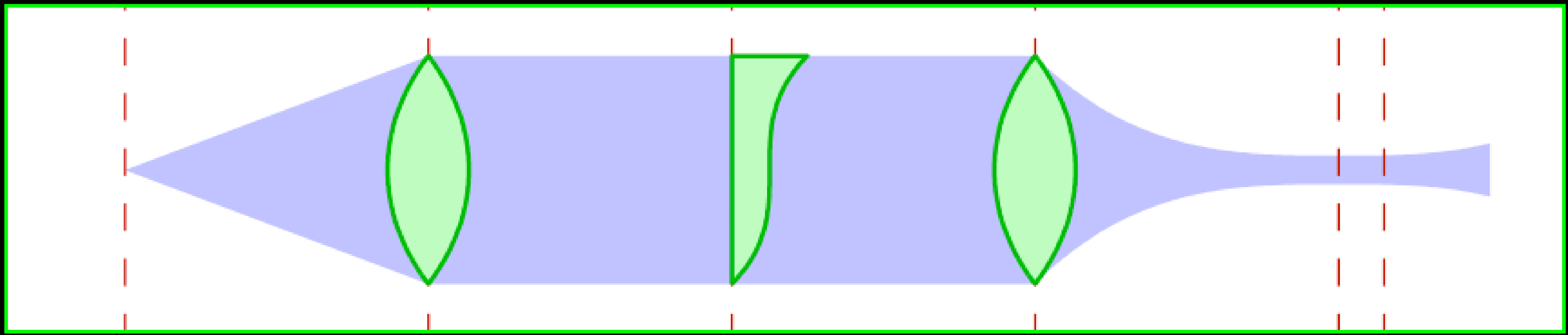
Wigner distribution

Papoulis 1974

ambiguity function

Application - Wavefront Coding

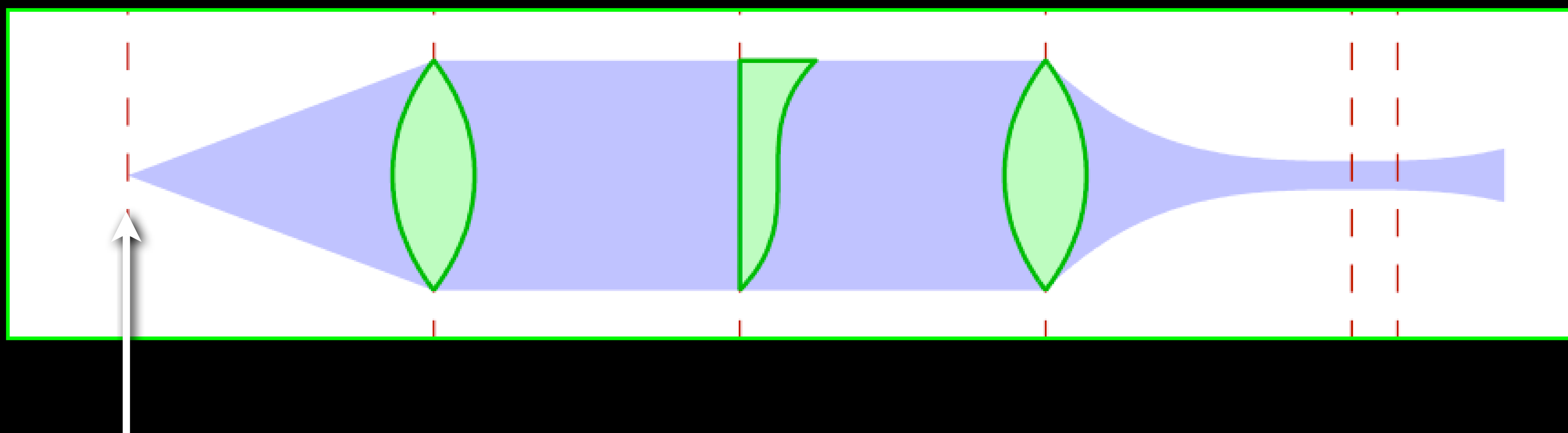
Dowski and Cathey 1995



same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995

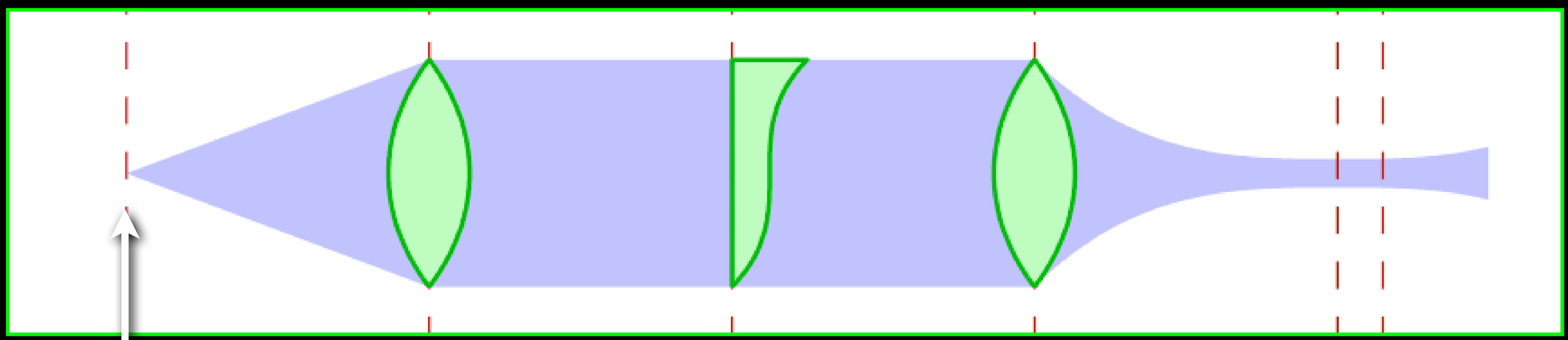


point
in scene

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995



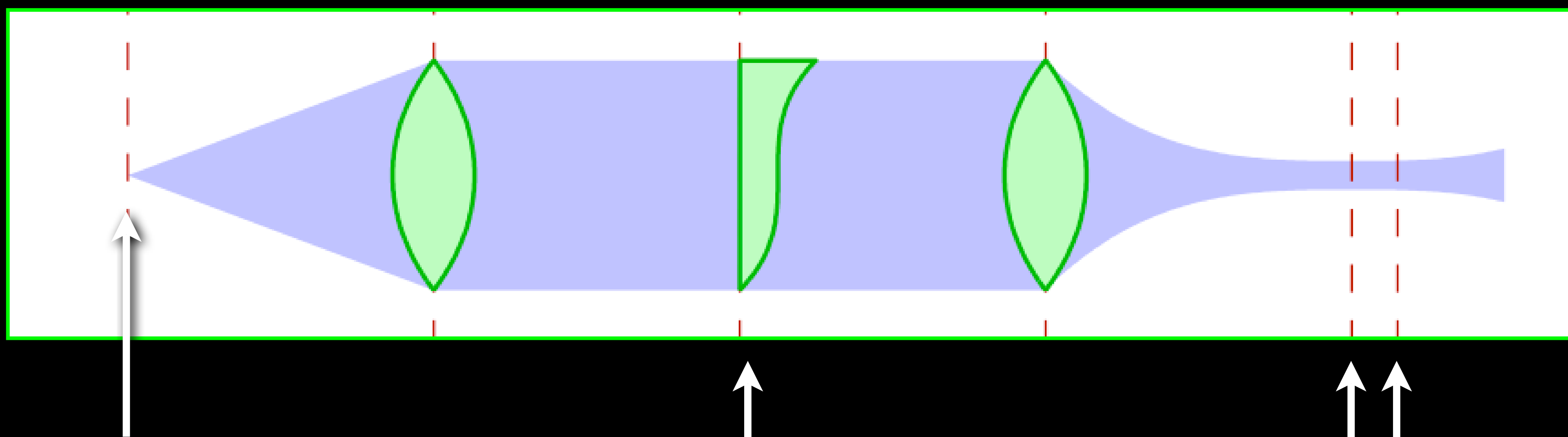
point
in scene

cubic
phase plate

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995



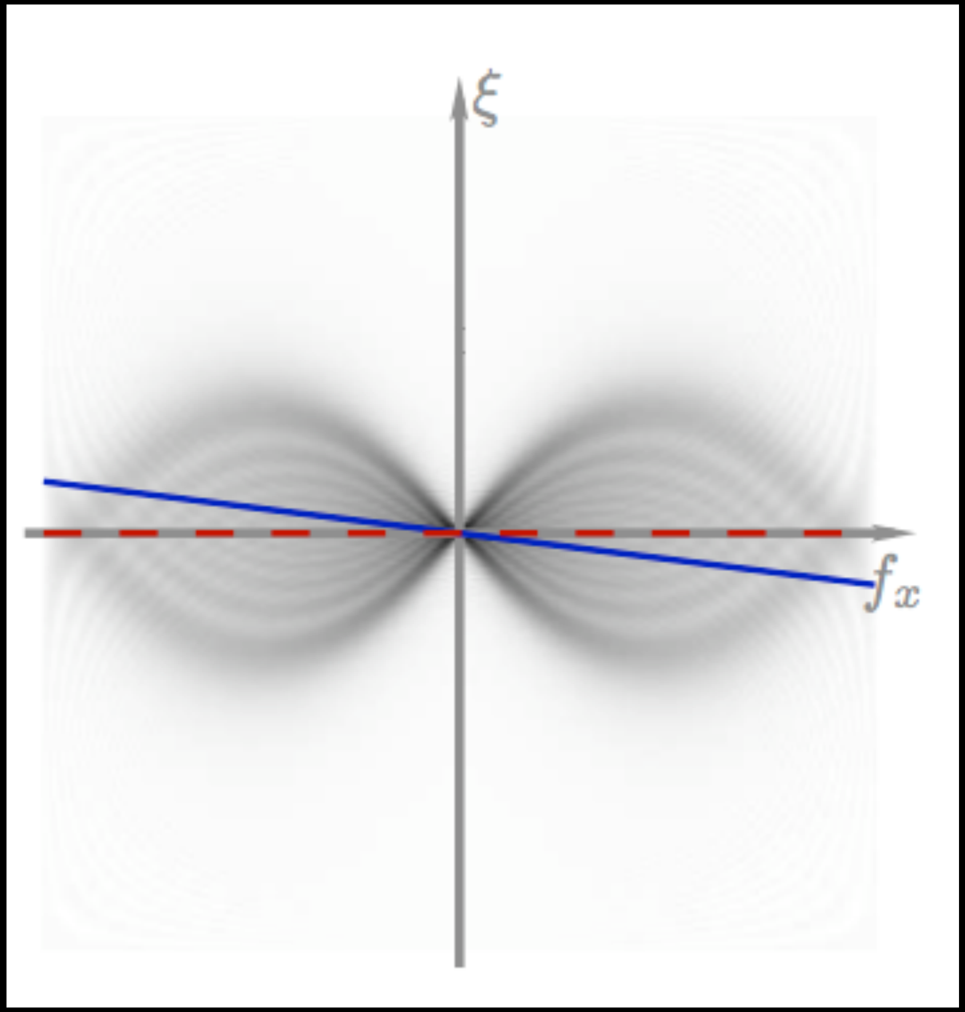
point
in scene

cubic
phase plate

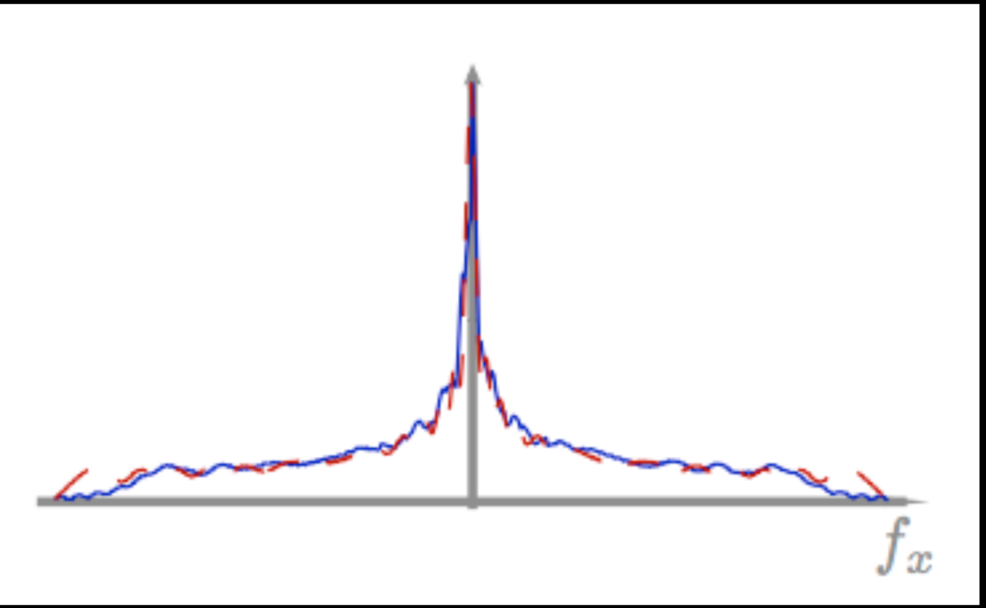
small change
in blur shape

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

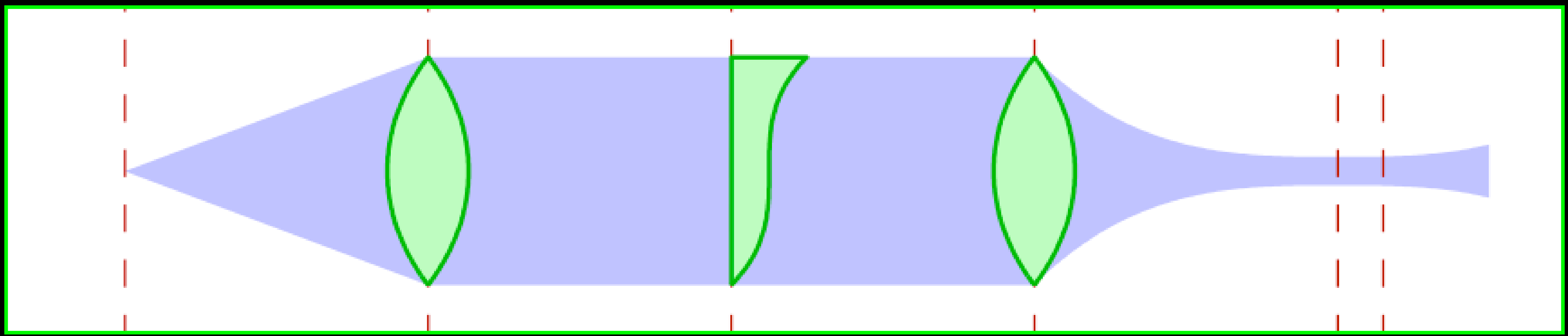


ambiguity function

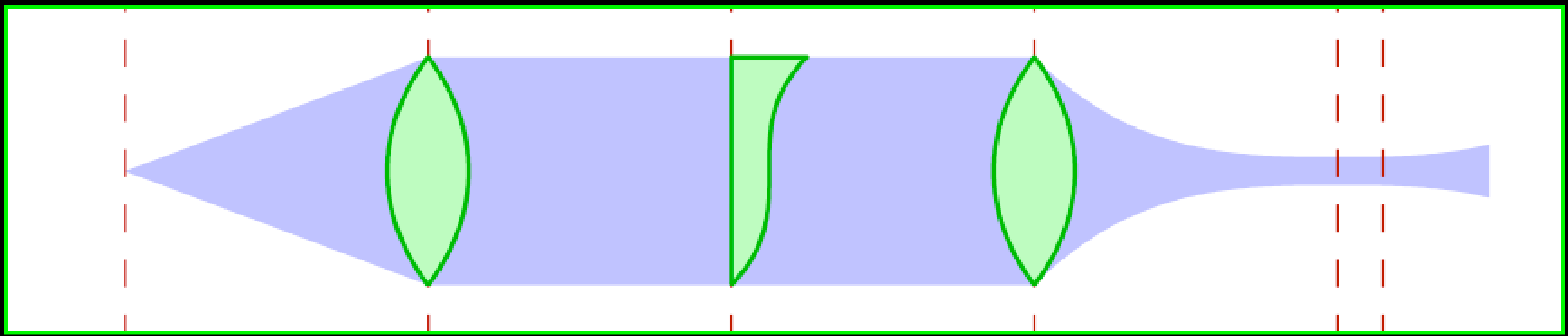


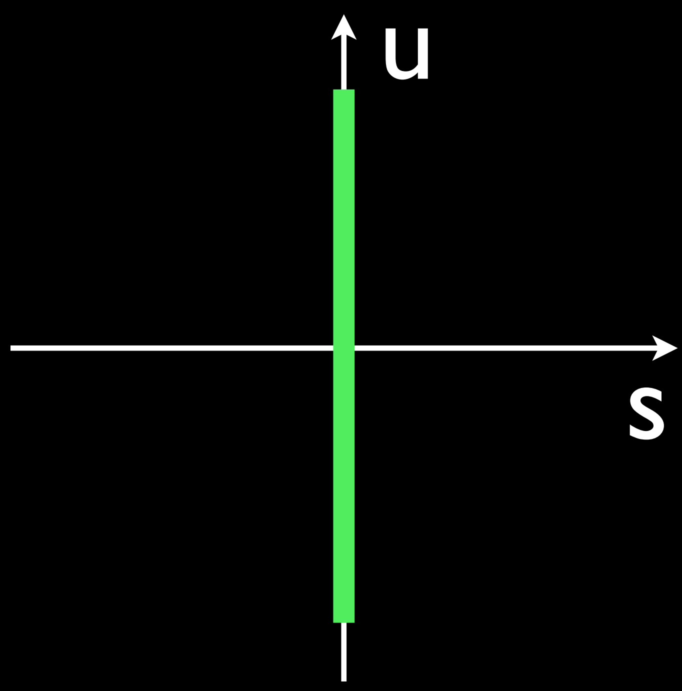
slices corresponding to various depths

Application - Wavefront Coding



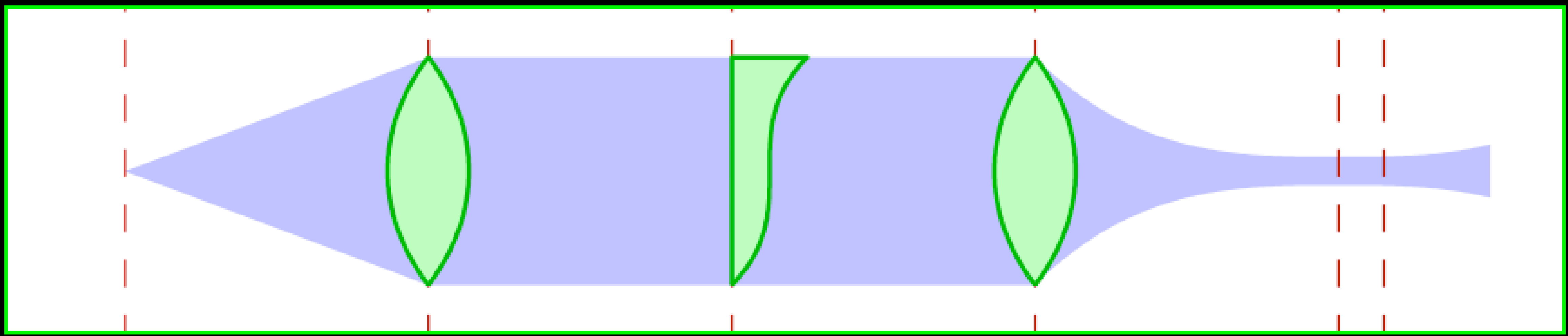
Application - Wavefront Coding

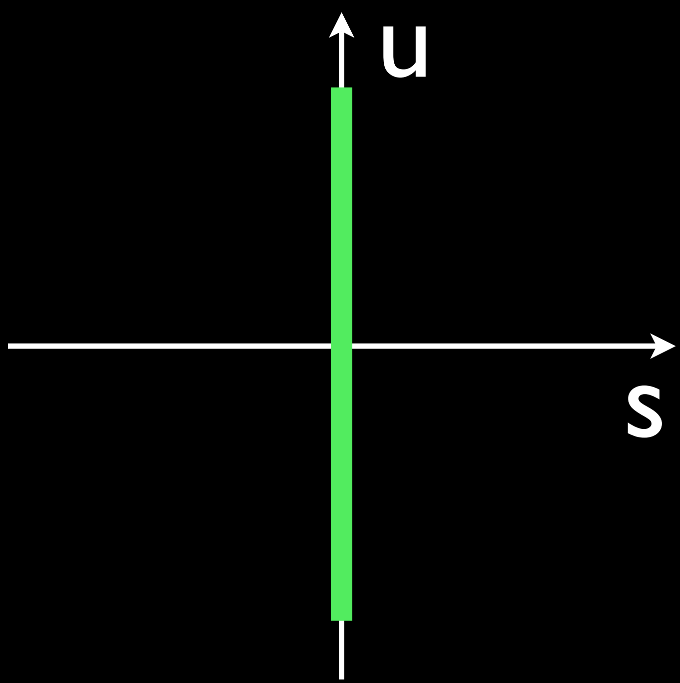




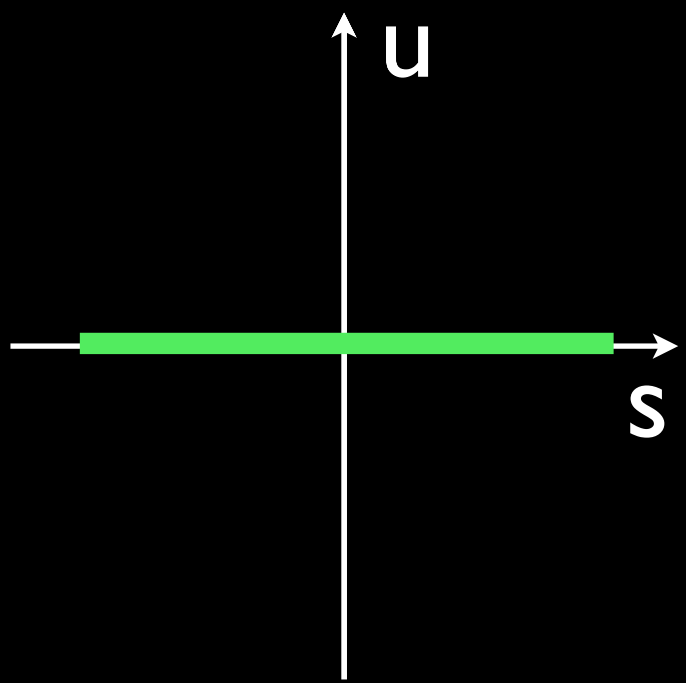
point

Application - Wavefront Coding



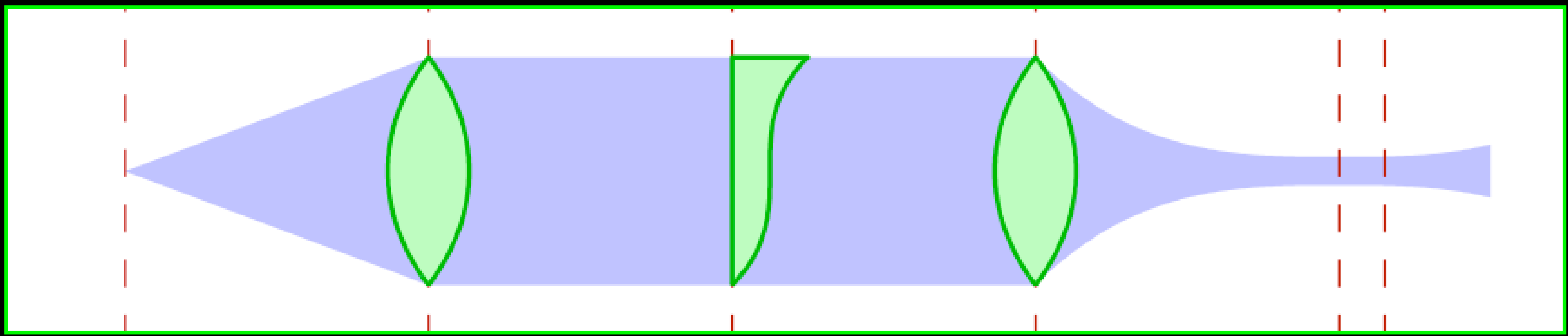


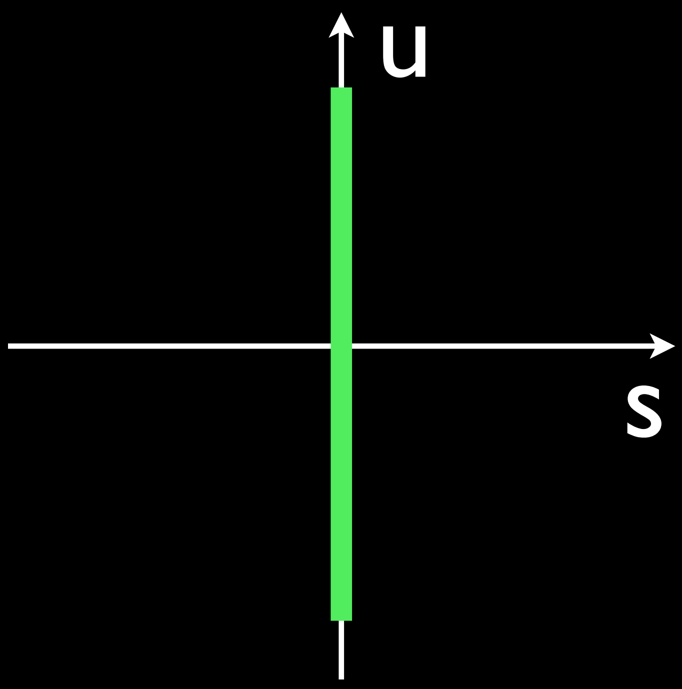
point



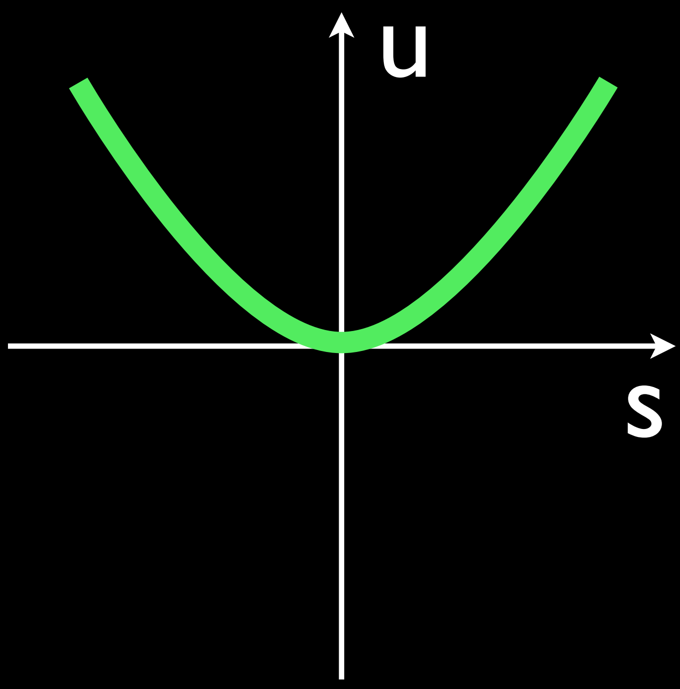
before phase plate

Application - Wavefront Coding



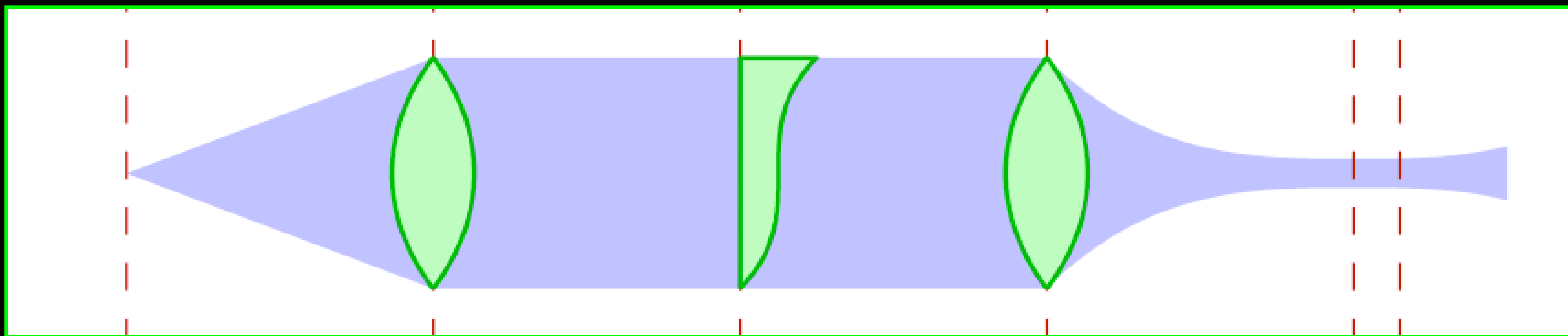


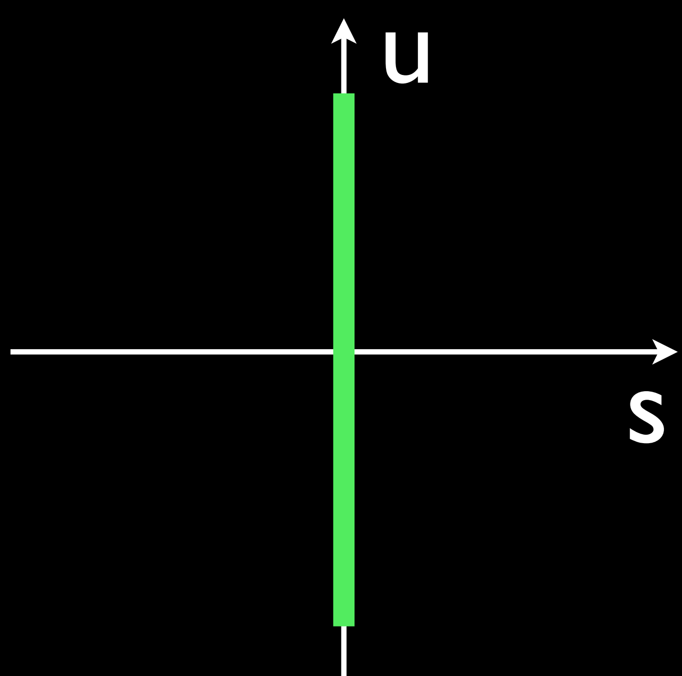
point



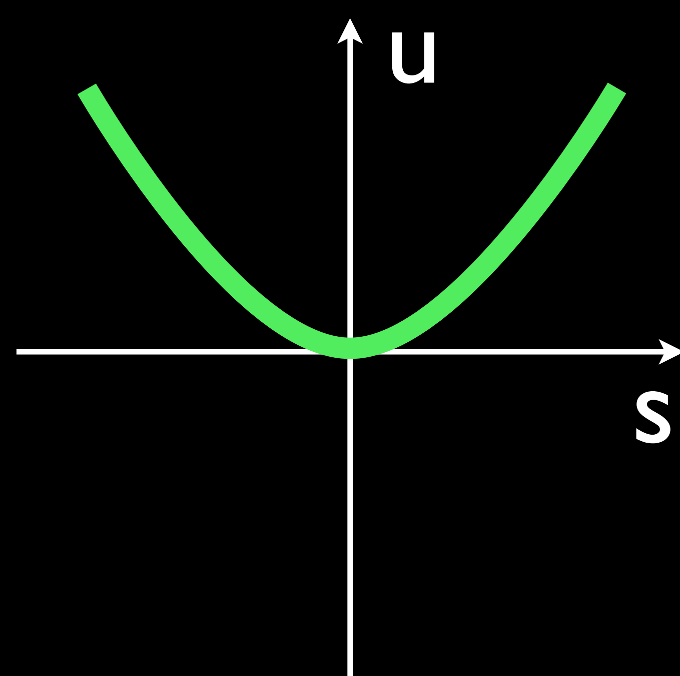
after phase plate

Application - Wavefront Coding

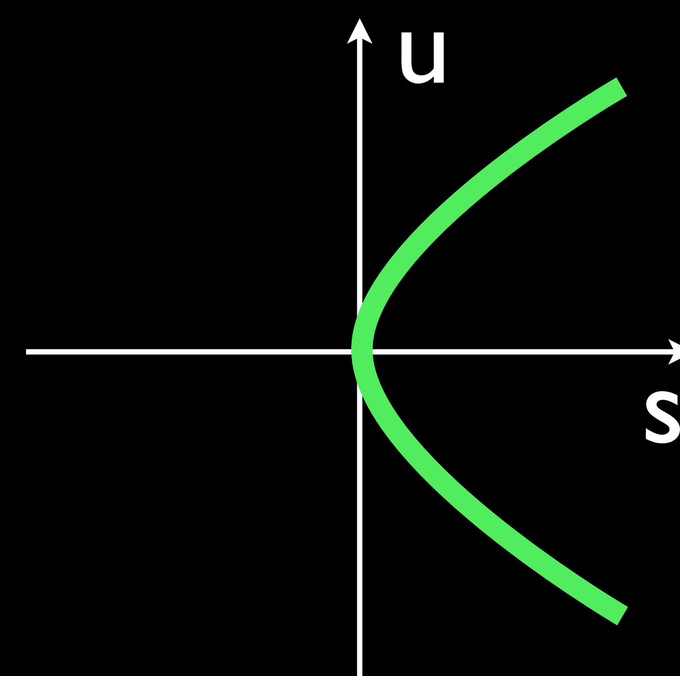




point

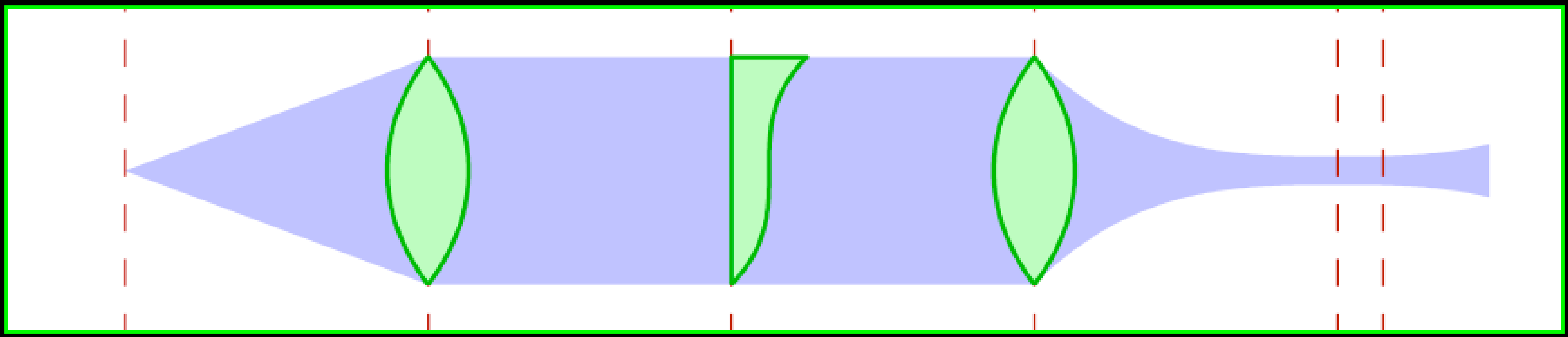


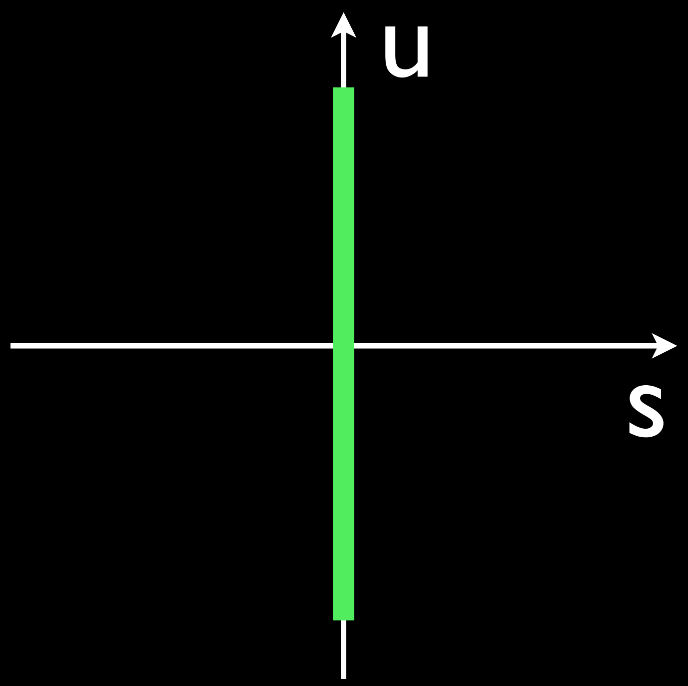
after phase plate



at image plane

Application - Wavefront Coding





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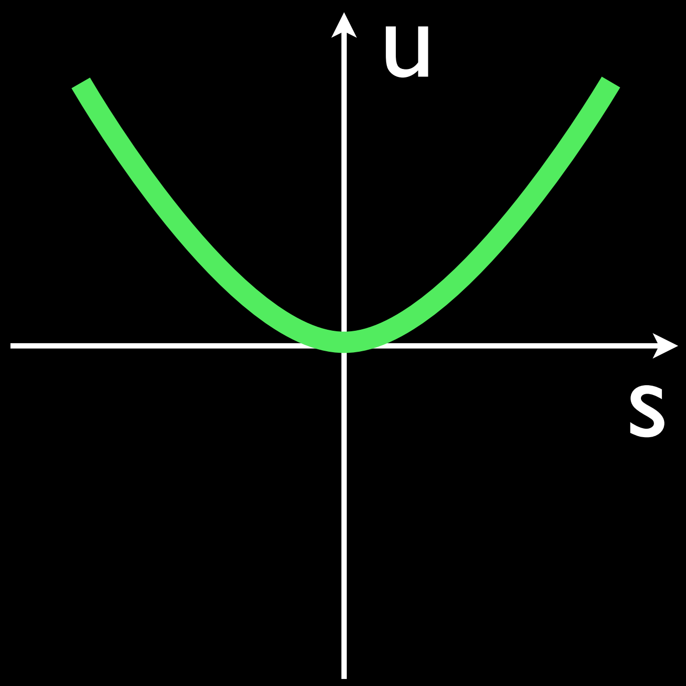
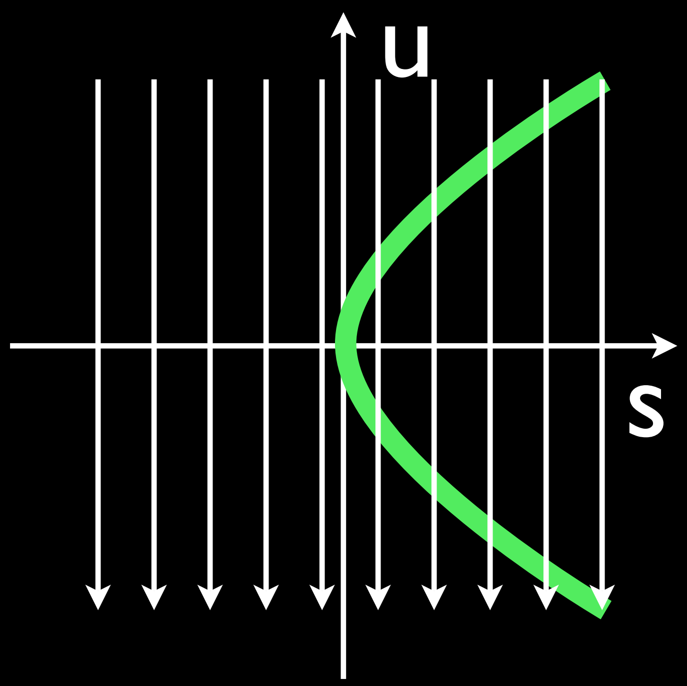


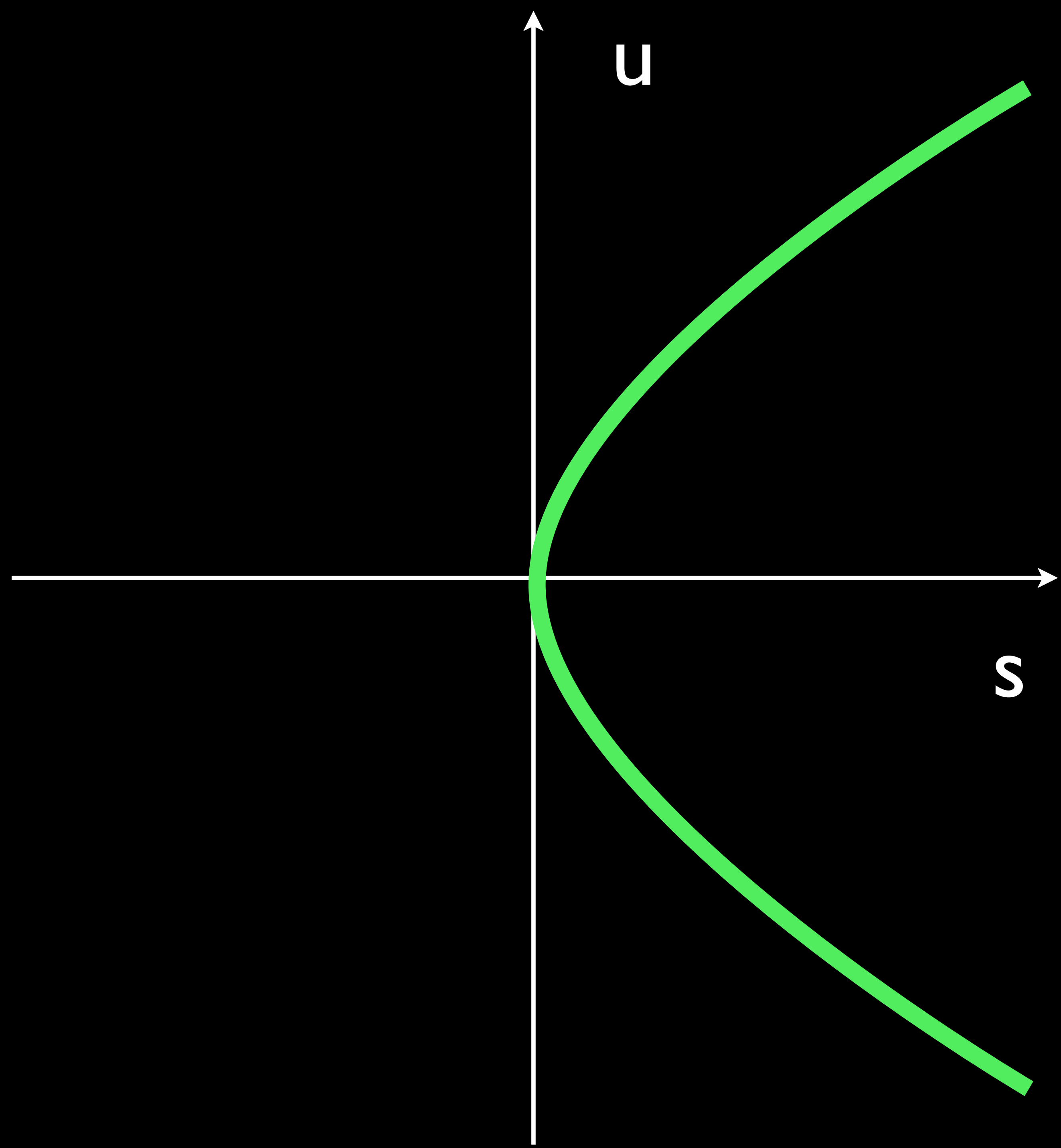
after phase plate



at image plane

Application - Wavefront Coding

- refocusing in ray space is shearing
- shearing of a parabola results in translation
- blur shape invariant to refocusing



Application - Wavefront Coding

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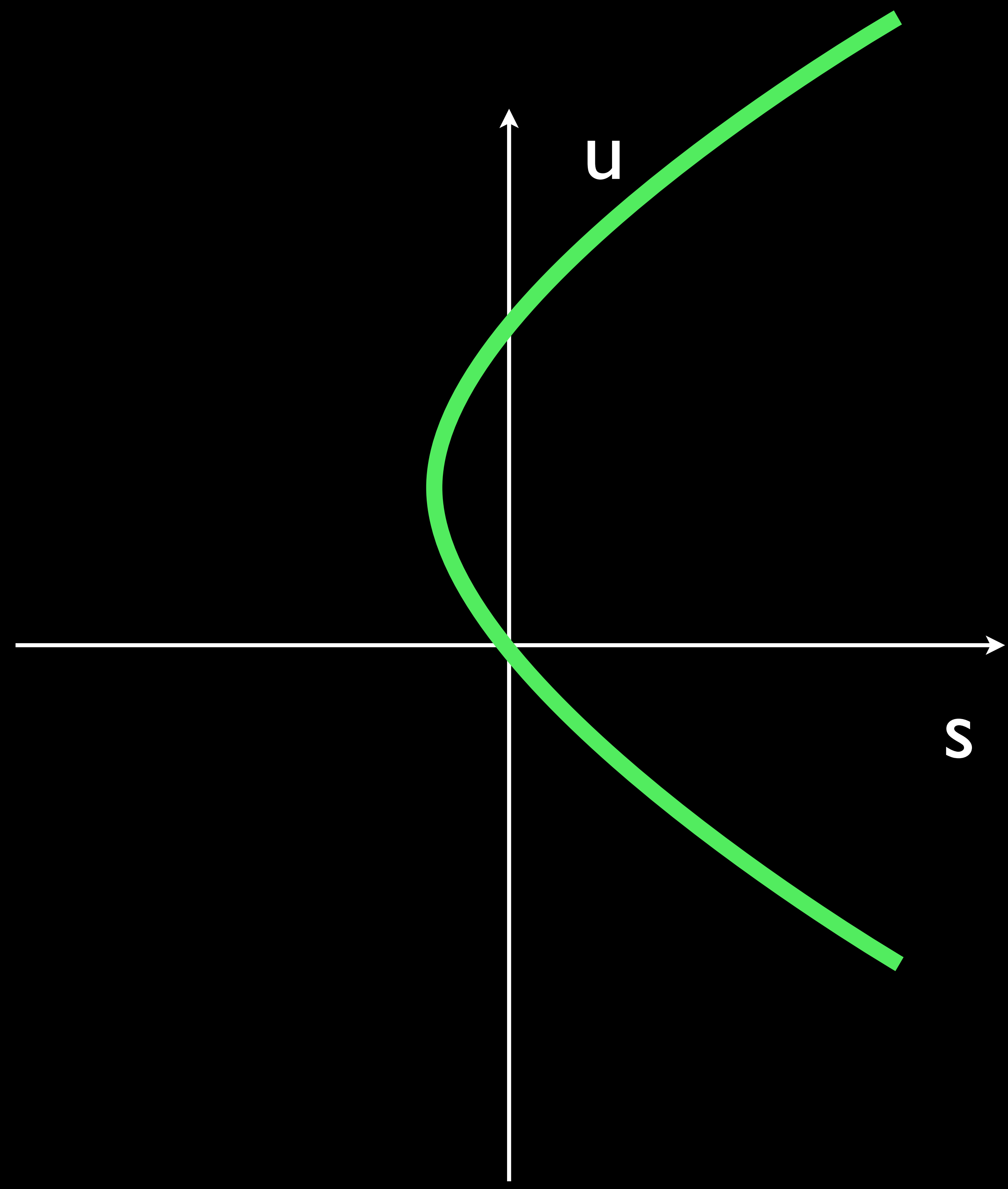
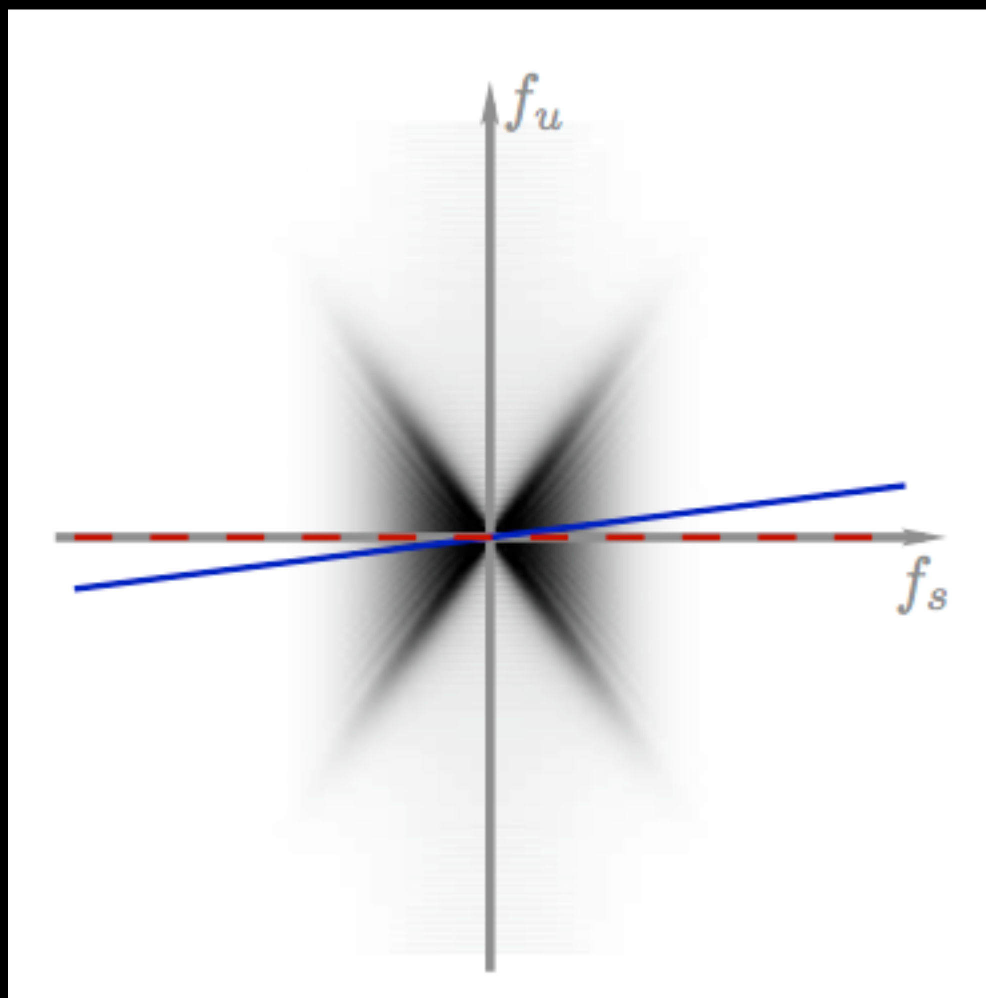
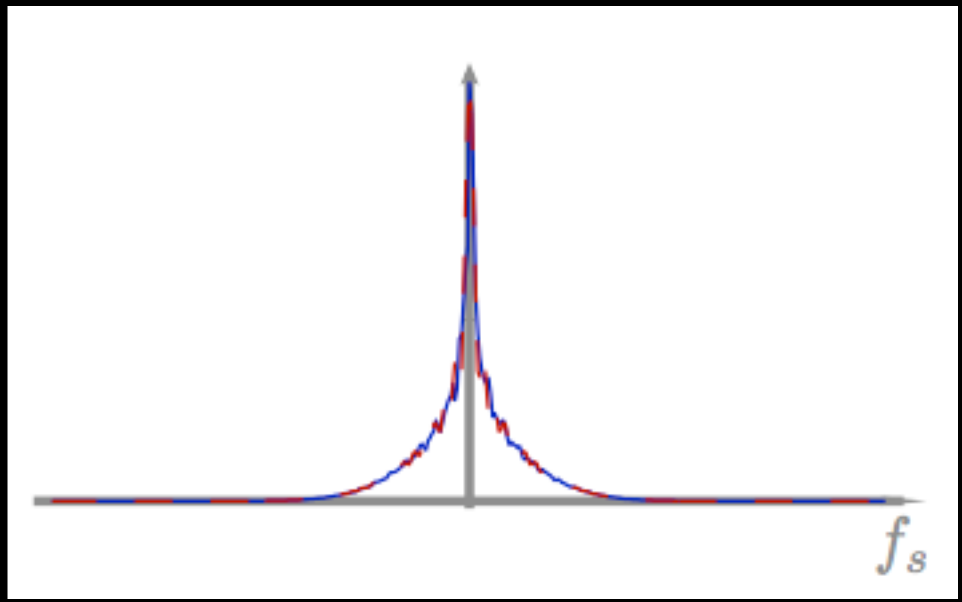


Application - Wavefront Coding



Fourier transform
of light field



slices corresponding
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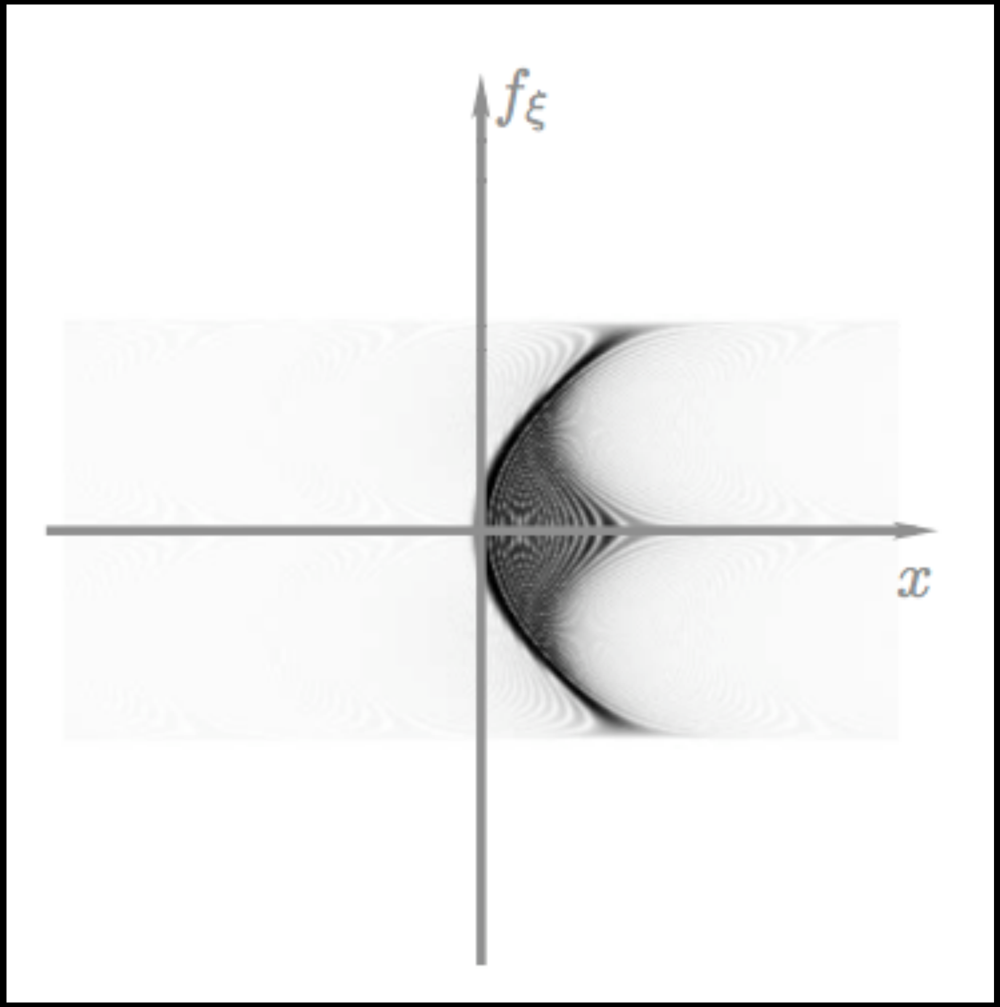
Application - Wavefront Coding



Wigner distribution
for cubic phase plate system

Conclusions

- light field's position and direction = wave optics's position and frequency
- observable light field = blurred Wigner distribution (equal at zero wavelength limit)
- analysis using light fields and Wigner distribution interchangeable

Further Reading

- http://scripts.mit.edu/~raskar/lightfields/

  Wiki for this course

- Z. Zhang, M. Levoy, "Wigner Distributions and How They Relate to the Light Field", ICCP 2009

Future Work

- analyze various light field capture and generation systems using wave optics
- rendering wave optics phenomena
- **adapt more ideas from optics community and vice versa!**

Acknowledgements

- Anat Levin, Fredo Durand and Bill Freeman
- Stanford Graduate Fellowship from Texas Instruments and NSF Grant CCF-0540872

Light Fields in Ray and Wave Optics

Introduction to Light Fields:	Ramesh Raskar
Wigner Distribution Function to explain Light Fields:	Zhengyun Zhang
Augmenting LF to explain Wigner Distribution Function:	Se Baek Oh
Q&A	
Break	
Light Fields with Coherent Light:	Anthony Accardi
New Opportunities and Applications:	Raskar and Oh
Q&A:	All